BASIC LOGICAL REASONING

BASIC MATHEMATICAL REASONING

AND

THE PHYSICAL SCIENCES

by

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PREFACE

This course provides a way for non-science freshman students to learn basic reasoning skills and appreciate their use in science. The approach strengthens the "liberal arts" notion of education by integrating logic, basic mathematics and some concepts from physical science into one course of study. In addition, there is practice at synthesizing ideas and writing. The process and "everyday" nature of science is emphasized. The general philosophy is to use physical science as a medium to help students develop logical reasoning, critical thinking and problem solving abilities.

The intent of this book is to provide a basic framework for a more detailed discussion among students, facilitated by an instructor. The "answers" are not found in these pages, but there is some basic background material that can be used as a reference.

Further explanation of this material, practice quizzes, and other details specific to PHY101 are found on the Introduction to Physical Sciences homepage which can be accessed through the Chemistry/Physics Department Access Page (http://www.rivier.edu/chemistry).

The homepage also has information about concept outlines, writing assignments, and oral presentations.

This class will not cover very much formal physical science. It will concentrate on basic skills and the processes that scientists use. Physical science will be used as a medium to develop logical reasoning and basic math skills. This is intended as a course that integrates and provides opportunities to develop general skills that can be used throughout life.

We will begin by looking at the atom so that an understanding of charge can be developed. Basic logic will be used to investigate the implications that arise from the commonly known concept that like charges repel and unlike charges attract. This will lead to some elementary chemistry and physics that can be applied to the world around us and help us to see the need to understand some basic science when dealing with the environmental concerns of today. After this you will be in a position to investigate one of the most important concepts in chemistry, the concept of intermolecular forces. Some elementary classical physics will then be investigated. The idea of forces causing motion and opposing forces balancing will be ongoing concepts. Conservation of energy, specifically dealing with kinetic and potential energy will be studied in more detail.

This book is intended as a supplement to class discussion and activities that will cover this material. It is not intended to encompass all details about these topics and, furthermore, other topics will also be part of the course.

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I. Introduction

A. Counter-Intuitive Situations

One reason the world is such a fascinating place is that something is always surprising us. This gives variety and excitement to our lives. But why are we surprised? Maybe it is due to experience. Gravity never changes, yet hot air balloons rise, airplanes take off, rockets propel men and women into space, and space probes leave the earth and travel beyond the limits of our solar system. These are not surprising to anyone that lives in our day, but can you imagine how surprised people who lived 1000 years ago would have been to find out that someone walked on the moon? How would they have explained such a feat? It would not have made any sense according to their explanation of the world, but in our day it can be completely explained by the "laws of physics." An event that goes against what we think should happen is said to be counter-intuitive.

Here is one for you to try. After Thanksgiving dinner (or at some other convenient time) connect the tines of two forks together, put one end of a toothpick through the tines, and place the other end of the toothpick on a drinking glass. If done correctly the forks will "hang in mid-air!" Figure 1.1 is a model of the situation.

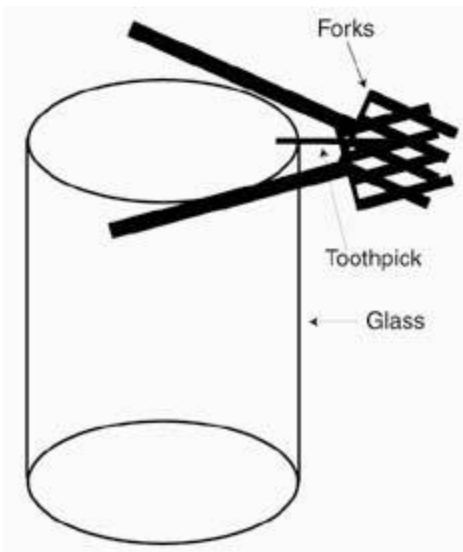


Figure 1.1

Isn't that fun! Some people think physics is hard because it is counter-intuitive, but the counter-intuitive situation is what makes physics interesting. One of our jobs is to understand the counter-intuitive situation. How do you go about understanding something that seems impossible?

B. Modeling

One way to try and make sense out of the insensible is to model it. Figure 1.1 is a model. It has been constructed from experience and can be used by others to duplicate the situation. Is this model useful? Can you make a better model? Is there another kind of model that would help us understand this situation better? Here is a picture of this situation. Is it a better model?

Modeling is central to a scientist's activities. It might be said that a scientist's goal is to correctly model observable events. The models can take several forms. This book will emphasize logical reasoning models as taught in beginning philosophy courses (see chapter two) and basic mathematical reasoning models as taught in Junior High and High Schools (see chapter three). Building models, using logical reasoning and mathematical reasoning, should become a skill that is applied to all problem solving, whether in science or any other discipline. Appendix A has a summary of some of the kinds of models that might be used in science.

C. A Good Learner

In order to understand counter-intuitive situations it is important to be a good learner. One important concept that must be learned is that scientists are ordinary people who use ordinary methods to solve problems. They must often learn new ideas and use those ideas in solving the problem at hand. It is important to improve learning skills. Appendix B is a model of a good learner. Good learners seem to have the characteristics listed in Appendix B. These are characteristics that should be developed through the study of science. Study Appendix B and decide what you think are the three most important characteristics for a good learner. Use the activities suggested in this book to develop those characteristics.

D. The Nature of Science

Science is inherently experimental and, as such, inherently has uncertainty associated with it. Scientists model physical observations using the best tools available. They are constantly trying to refine their understanding of how nature works. They are in a never-ending quest for truth. Consider the following quotes from Daniel Dennett in his article *Postmodernism and Truth* (http://www.butterfliesandwheels.com/articleprint.php?num=13).

"Then we invented measuring, and arithmetic, and maps, and writing. These communicative and recording innovations come with a built-in ideal: truth. The point of asking questions is to find true answers; the point of measuring is to measure accurately; the point of making maps is to find your way to your destination."

"We human beings use our communicative skills not just for truth-telling, but also for promise-making, threatening, bargaining, story-telling, entertaining, mystifying, inducing hypnotic trances, and just plain kidding around, but prince of these activities is truth-telling, and for this activity we have invented ever better tools. Alongside our tools for agriculture, building, warfare, and transportation, we have created a technology of truth: science. Try to draw a straight line, or a circle, "freehand." Unless you have considerable artistic talent, the result will not be impressive. With a straight edge and a compass, on the other hand, you can practically eliminate the sources of human variability and get a nice clean, objective result, the same every time."

"What inspires faith in arithmetic is the fact that hundreds of scribblers, working independently on the same problem, will all arrive at the same answer (except for those negligible few whose errors can be found and identified to the mutual satisfaction of all). This unrivalled objectivity is also found in geometry and the other branches of mathematics, which since antiquity have been the very model of certain knowledge set against the world of flux and controversy."

"Yes, but science almost never looks as uncontroversial, as cut-and-dried, as arithmetic. Indeed rival scientific factions often engage in propaganda battles as ferocious as anything to be found in politics, or even in religious conflict. The fury with which the defenders of scientific orthodoxy often defend their doctrines against the heretics is probably unmatched in other arenas of human rhetorical combat. These competitions for allegiance--and, of course, funding--are designed to capture attention, and being well-designed, they typically succeed. This has the side effect that the warfare on the cutting edge of any science draws attention away from the huge uncontested background, the dull metal heft of the axe that gives the cutting edge its power. What goes without saying, during these heated disagreements, is an organized, encyclopedic collection of agreed-upon, humdrum scientific fact."

"The methods of science aren't foolproof, but they are indefinitely perfectible. Just as important: there is a tradition of criticism that enforces improvement whenever and wherever flaws are discovered. The methods of science, like everything else under the sun, are themselves objects of scientific scrutiny, as method becomes methodology, the analysis of methods. Methodology in turn falls under the gaze of epistemology, the investigation of investigation itself--nothing is off limits to scientific questioning."

The "technology of truth" called science often seems contradictory. It is inherently uncertain, but everyone can get the same result. It is always changing, but there is a "humdrum of scientific fact" that is agreed upon by all scientists. Maybe some explanations are in order.

Are There Facts in Science?

In science the words hypothesis, theory, law, and fact are often used. What is the difference between a hypothesis and a theory? Or between a theory and a law? Or between a law and a fact?

Here are some definitions that may be used.

- **Hypothesis** - An idea that could explain an interesting observation. It is limited in scope and is an educated guess, meaning that there is reasoning that supports the idea. A guess without any reason for the guess is just a guess, not a hypothesis!
- **Theory** - A theory is broader in scope, has experimental support, is coherent, and is internally consistent. Experimental support means that experimental observations are consistent with the theory and experimentation bears out the truthfulness of predictions made by the theory. To be coherent and internally consistent means that all aspects of the theory itself are consistent. One part of the theory doesn't contradict another part of the theory. In science we are interested in theories that explain how something works or how an observation came to be that way.
- **Law** - Laws can be thought of as theories that are so well established that they seem to always be true under the specified conditions. Newton's laws are about force and motion as applied to large objects. If a very small particle is traveling very fast (close to the speed of light) Newton's laws no longer hold true. This idea that there are conditions that must be met in order for the law to be true is often overlooked.
- **Fact** - Facts can be thought of as pieces of experimental data that are so well established that they seem to always be true under the specified conditions. The boiling point of water is 100^{o}C (a fact), but it is only true at one atmosphere of pressure. Science depends on the idea that the "facts" of science can be obtained by anyone anywhere as long as the conditions are the same. I emphasize that every fact will have a set of conditions attached to it.
- **Data** - Data comes from confirmed observations. A confirmed observation is a fact, it has been experimentally verified and is reproducible under the specified conditions. One piece of data is a fact.

Fundamental Principles

There is another category that is sometimes called fundamental principles. These fundamental principles have held true throughout all of the history of humankind. The laws of thermodynamics are in this category. Conservation laws are also in this category. One expression of the conservation laws is that you can't get something from nothing. That seems to be a principle that is always true.

These fundamental principles hold a special place when evaluating an idea as a scientific theory. No idea will have very much merit if it violates one of the fundamental principles that have been established over time.

II. Basic Logical Reasoning

". . . each chief step in science has been a lesson in logic."
A quote from Charles Sanders Pierce

A. Logic and Reasoning

One of the most important aspects of a scientist's work is the use of argument, presenting evidence to support conclusions. Scientists must argue logically in order to organize their observations into general, scientifically useful statements. They must argue logically in order to effectively support their theories. They must also argue logically in order to have their theories accepted by other members of the scientific community. Information, however accurate, which is not formulated completely and organized logically will be quickly dismissed. It is, therefore, important for the scientist, and the student of science, to be able to recognize arguments, understand how arguments are structured, and be able to determine whether arguments are effective or not.

It is also important to realize that the use of argument is not by any means confined to the scientific context. In fact, arguments are exchanged in all domains of life: in the various academic disciplines, in the workplace, in the law courts, in ordinary, daily conversation. Whenever we want to prove a point, to convince someone of an opinion held, and we provide information to support that point or opinion, then we are engaged in argument. In this sense, an argument is not a fight or disagreement, but rather an effort to reason with someone. In every argument information already known or agreed upon is used to prove a point not yet known or accepted. In an argument the known information is expressed as premise statement(s), and the point to be proved is the conclusion statement.

Some examples of arguments are now given to illustrate both how arguments work and the diversity of contexts in which they can occur:

1. The first argument is taken from The Trial, by Franz Kafka.

> *Someone must have been telling lies about Joseph K., for without having done anything wrong he was arrested one fine morning.*

In this argument from the novel by Kafka the speaker is drawing the conclusion that "someone must have been telling lies about Joseph K." As readers, we would assume, or know from the context in the novel, that the speaker is aware of Joseph K.'s arrest and also aware that Joseph K. had done nothing wrong. We would then see that, in light of this premise information, the speaker concludes that Joseph K. was a victim of lying.

2. The second argument is from an article by A. M. Turing, "Computing Machinery and Intelligence," Mind, Vol. 59, 1950.

> *Thinking is a function of man's immortal soul. God has given an immortal soul to every man and woman, but not to any other animal or to machines. Hence no animal or machine can think.*

In this argument A. M. Turing, the pioneering giant in the field of computers and artificial intelligence, is exploring the position that computers can not think. In order to make a case for that position he is appealing, in his premises, to information he believes his audience might well already accept about the nature of the human soul and the absence of a soul in machines and other animals. Notice how this argument is building. The first premise connects thinking with having an immortal soul. The second statement actually makes two points. It links an immortal soul with man and woman. It also excludes an immortal soul from animals and machines. This premise information leads then to the conclusion statement. In other words, Since thinking requires an immortal soul and animals and machines have no soul, we are led to conclude that "no animal or machine can think."

3. The third argument is from William Hochkammer, "The Capital Punishment Controversy," Journal of Criminal Law, Criminology, and Political Science, Vol. 60, No. 3, 1969.

> *But since the death penalty is in fact imposed for only those capital crimes which shock the public, where guilt is clear, and in light of the existing safeguards of appellate review and the possibility of commutation, execution of the innocent is unlikely.*

In this journal article Hochkammer is arguing, from information about how the death penalty is actually applied, to the conclusion that execution of the innocent is unlikely. Several points are included in the premise material: That the death penalty is actually imposed (1) only for capital crimes, (2) only for those capital crimes that shock the public, (3) only in cases where guilt is clear, and (4) only in the context of the existing legal safeguards of appellate review and the possibility of commutation. From this premise information Hochkammer draws the conclusion that "execution of the innocent is unlikely."

4. The last example is from Robert Heilbroner, "Reflections: Boom or Crash," The New Yorker, August 28, 1978.

> *Human activity, especially in the industrialized regions where its effect on nature is most concentrated, is at the verge of creating violent and irreversible effects on the planet. The immense magnitude of technological assault on the environment is indicated by the ongoing debates over the possibility of carbon dioxide creating a "greenhouse effect" that would alter the temperature of the entire earth; the possibility that massed industrial heat could change the patterns of air circulation and of precipitation on a continental scale; the possibility that the release of chemical waste might contaminate the ground water of a large region; and the possibility that the volume of nuclear wastes might constitute a hazard for an entire city or state.*

In this argument economist Robert Heilbroner is drawing on four major points of information to establish and support his conclusion, which is expressed in the first statement. His premises are based on observations about (1) the possibility of the "greenhouse effect," (2) potential changes in the patterns of air circulation and precipitation, (3) the possibility of widespread chemical contamination of ground water, and (4) the potential dangers of nuclear wastes. In light of these serious dangers from industrial activity, Heilbroner asks us to conclude that we are "at the verge of creating violent and irreversible effects on the planet."

B. Argument and Logical Form

Arguments can be looked at in various ways. We can notice their content, what the statements are about. We can also pay attention to their form or structure, how they are set up and the relationships among the various statements. We can try to determine how well the premise statement(s) support the conclusion statement. And we can ask whether the premise statements are true or false. In fact, for a critical interpretation and evaluation of any particular argument we must do all of these.

Questions about how well the premises support the conclusion are often questions primarily about the form or structure of the argument. Thus, it is important to be able to recognize and work with different argument forms.

As an example take the following argument, drawn from Meno, a dialogue by the ancient Greek philosopher, Plato:

> *If virtue is a kind of knowledge, then it can be taught. But virtue is a kind of knowledge. Therefore, virtue can be taught.*

This is a type of argument that has a very specific logical structure. Its form or structure can be clearly seen by re-writing the argument in standard form, identifying the premises (labeled "P") and conclusion (labeled "C") in the following manner:

> P1 --- If virtue is a kind of knowledge, then it can be taught.
>
> P2 --- virtue is a kind of knowledge.
>
> C --- virtue can be taught.

Perhaps you can already begin to see the logical form or structure of this argument. The first premise in this argument is a particular kind of compound statement called a conditional statement. It has two parts: the antecedent, which comes after the word "if" and expresses a condition, and the consequent, after the word "then," which expresses what will be true, if the condition expressed in the antecedent is met. In this argument the second premise affirms that the antecedent condition has been met. From these two premise statements I may now infer or draw the conclusion that virtue can be taught. The following analysis and labeling of the argument highlights these points.

[antecedent]   [consequent]

P1 --- If virtue is a kind of knowledge, then it can be taught.

P2 --- virtue is a kind of knowledge. (affirms the antecedent)

C --- virtue can be taught. (affirms the consequent)

This argument is valid. That is, the premise statements successfully support the conclusion statement to such an extent that if these two premise statements are true, then the conclusion statement has to be true. The argument has very strong reasoning.

What is interesting and important about this example, however, over and above what it seeks to prove about the teachability of virtue, is its logical form or structure. It is worth noting that any argument, about any subject matter, provided it has the same logical form or structure, will also be a valid argument.

Consider the following argument:

If I travel for one hour at fifty-five miles per hour, then I will travel fifty-five miles. I did travel for one hour at fifty-five miles per hour. Thus, I traveled fifty-five miles.

In standard form this will be as follows:

[antecedent]   [consequent]

P1 -- If I travel for one hour at fifty-five miles per hour, then I will travel fifty-five miles.

P2 -- I did travel for one hour at fifty-five miles per hour. (affirms the antecedent)

C -- I traveled fifty-five miles. (affirms the consequent)

This argument, and any argument having the same logical form, will be a valid argument, with very strong reasoning between premise(s) and conclusion.

Conditional arguments are only one of many different types of argument that can be analyzed and evaluated from a consideration of their logical form. It is highly useful to become acquainted with the different forms of argument, not only the forms that are valid, but also the forms that are invalid.

C. Other Types of Argument

Some arguments work or fail to work, not merely because of their logical form, but due to other kinds of relations among premise(s) and conclusion.

Consider the following argument from an article in the May 1992 edition of Discover Magazine, titled "Ruffled Feathers" by Carl Zimmer, which is based on analogy, similarity and dissimilarity:

> *"In 1983 Chatterjee found some bones, . . . [As he pieced them together] he began to notice some odd things. The shoulder bone, for instance, was much longer than those of most dinosaurs but a lot like those of modern birds. And the neck vertebrae had a saddle shape, one peculiar for dinosaurs but normal for birds --- it makes their necks flexible.*
>
> *"As Chatterjee assembled the shattered bits of skull, he found something that for him was even more striking. Behind the eye of a dinosaur are two holes in its skull, divided by a bony strut. In the course of developing a more flexible jaw, birds have lost this strut. 'I noticed there was just one hole,' says Chatterjee. 'This is the most distinctive feature of the bird skull.' He believed he could even see little knobs in the arms of the skeleton where feathers would have been rooted. By the end of 1985 he thought there was a pretty good chance that he had actually found a bird."*

Put into standard form, this argument looks like this:

P1 The shoulder bone was much longer than those of most dinosaurs, but a lot like those of modern birds.

P2 The neck vertebrae had a saddle shape, one peculiar for dinosaurs, but normal for birds (making their necks flexible).

P3 In the skull, behind the eye, there was just one hole. Behind the eye of a dinosaur, there are two holes in the skull, divided by a bony strut. In the course of developing a more flexible jaw, birds have lost this strut; the presence of a single hole being the most distinctive feature of the bird skull.

P4 Chatterjee also believed that he could see little knobs in the arms of the skeleton where feathers would have been rooted.

C The bones Chatterjee found were not those of a dinosaur, but rather those of a bird.

This argument combines the use of analogy and disanalogy, similarity and dissimilarity, to draw a conclusion about the bones that were discovered, in order to determine to what kind of animal they belonged.

A strong literal analogy, sufficient for drawing a conclusion, exists between two or more things when they share a number of important and relevant characteristics in common. In the above example, the identified skeletal structures (length of the shoulder bone, shape of the neck vertebrae, etc.) are important and relevant characteristics for identifying and distinguishing different types of animals. Since observation of the bones revealed several (in this case four) relevant and important points of similarity to the modern bird, there is good reason to conclude that the bones were those of a bird. There is, of course, no way of knowing with certainty that this is the case, yet the strong analogy makes the conclusion very likely to be true. This kind of argument is based on a logical principle called the "principle of analogy": Things observed to have a number of closely similar properties in respects relevant to the comparison, most likely have other, unobserved properties in common as well.

The argument presented above also establishes a strong disanalogy between the bones found and those of dinosaurs. Once again the comparison is made in terms of important, relevant characteristics for identifying and distinguishing animals. In this case, the number of relevant dissimilarities establishes a disanalogy, supporting the other part of the conclusion, i.e. that the bones were not those of a dinosaur. The logical idea here is that things observed to have a number of dissimilar properties in respects relevant to the comparison are more likely to be dissimilar in other respects as well.

The following argument is a <u>causal</u> argument. The conclusion is that the diet of polished, white rice is what caused the chickens to develop polyneuritis and die.

> *Eijkman fed a group of chickens exclusively on white rice. They all developed polyneuritis and died. He fed another group of foul unpolished rice. Not a single one of them contracted the disease. Then he gathered up the polishings from rice and fed them to other polyneuritic chickens, and in a short time the birds recovered. He had accurately traced the cause of polyneuritis to a faulty diet. For the first time in history, he had produced a food deficiency disease experimentally, and had actually cured it. It was a fine piece of work and resulted in some immediate remedial measures.*
>
> ------ Bernard Jaffe, Outposts of Science (1935)

This argument could be written in the following way:

P1 Eijkman fed a group of chickens exclusively on white rice.

P2 They all developed polyneuritis and died.

P3 He fed another group of foul unpolished rice.

P4 Not a single one of them contracted the disease.

P5 He gathered up the polishings and fed them to other polyneuritic chickens.

P6 In a short time the birds recovered.

C The diet of polished, white rice is what caused the chickens to develop polyneuritis and die.

D. Complex Arguments

Some arguments are simple inferences, in which any number of premises are offered to support one overall conclusion. All the arguments used above are simple inferences.

Other arguments have a complex structure. In these arguments one or more premises are offered to support a conclusion statement, which is itself then used as a premise to support a further conclusion. In these cases an argument is a chain of reasoning. Each link of the chain is a simple inference (simple argument), but, when strung together, they make a complex argument chain. Such an argument has what are called basic premises (BP), an intermediate conclusion (IC), which is also called a non-basic premise (NBP) in light of the fact that it functions as the conclusion of one inference, but then also as a premise for the next inference, and one final, overall conclusion (C).

Consider the Rutherford scattering experiment, which gives rise to a complex argument. During the time of Rutherford the raisin pudding model of the atom was popular. In this model very small electrons were "sprinkled" around in a positive charge that had no region of concentrated mass. This was like raisins (electrons) in a pudding (the positive charge). Rutherford knew about relatively large alpha particles and decided to test the raisin pudding model. He shot alpha particles at gold atoms and watched where the alpha particles went. He thought they should go straight through because, according to the raisin pudding model, there was no concentrated mass to deflect them. Rutherford knew that usually the only way for an object to be deflected is for it to collide with a larger object. The electrons were known to be too small to deflect the larger alpha particles. To his surprise the alpha particles were deflected in all directions, some of them even bouncing back toward the source of alpha particles! What argument might Rutherford write?

(1) BP --------- An alpha particle is deflected from its path only as a result of a collision with a solid object larger than itself.

(2) BP -------- Alpha particles shot into a gold atom are deflected.

(3) IC/NBP --- Gold atoms contain particles larger than alpha particles.

(4) BP --------- Electrons are much smaller than alpha particles.

(5) C --------- Gold atoms must have another particle besides the electron.

In this argument statements #1 and #2 are basic premises supporting the conclusion in statement #3. Statement #3, however, is not the final, overall conclusion. That is why it is labeled as an intermediate conclusion (IC). Statement #3 is also functioning as a premise, working with statement #4, to support the overall conclusion of the argument, statement #5. That is why statement #3 is also labeled as a non-basic premise (NBP). It is functioning as the conclusion of the first inference (a simple argument) and as a premise in the second inference. Any argument with an IC/NBP is called a complex argument in order to recognize its structure as a chain of reasoning, consisting of two or more inferences interconnected in this fashion.

E. Truth and Falsity of Premises

As suggested above, the overall purpose of an argument is to prove that its conclusion is true. In order for an argument to be successful at this, it must meet two criteria:

1. the reasoning or logical connection between premise(s) and conclusion must be sufficiently strong;

2. the premise(s) must be true.

Only if both criteria are satisfied does the argument give us good reason to believe that the conclusion is true. When both criteria are satisfied the argument leads to the truthfulness of the conclusion and it is said to be a sound argument.

Consider the following argument:

P1 --- If an acid is placed in water, then the OH^- concentration of the solution is increased.

P2 --- HCl is an acid.

C --- If HCl is placed in water, then the OH^- concentration of the solution will be increased.

This is a valid argument. Its premises support the conclusion such that if the premises are true, the conclusion will have to be true. The reasoning from premises to conclusion is very strong. However, the argument fails overall to prove that the conclusion is true because a premise, the first premise, is false.

The following argument is a fully successful argument because its reasoning is strong and all its premises are true. This argument does lead to the truthfulness of the conclusion.

P1 --- If an acid is placed in water then the H^+ concentration of the solution is increased.

P2 --- HCl is an acid.

C --- If HCl is placed in water then the H^+ concentration of the solution will be increased.

The principal job of logic is to help us understand under what conditions the reasoning of arguments is strong or weak, whether the premise statements do or do not support the conclusion. Logic does not help us to know whether, for example, the above statements about acids are true or false. One of the objectives of this course is to build up a store of knowledge as a basis for determining the truth or falsity of such statements. A second, and more important, objective of this course is to provide you with some general concepts and principles and to give you practice with the process of science. This should allow you to look at the premises (sometimes called theories, laws, hypotheses, etc.) in a more sophisticated way, in a way that allows you to better gage the truthfulness of those premises. It should by now be clear why both logical reasoning and truthful statements (or at least statements that agree with all of the available information from experiments, theoretical considerations, etc.) are necessary for the ongoing work of science.

In summary it should be noted how each piece of evidence (each premise) is directly related to a conclusion in every argument. This structure of substantiating a conclusion statement (hypothesis, opinion, etc.) is critical to scientists. In addition, the statements used as premises must agree with currently held concepts and principles (they must be "true").

Appendix C clarifies and expands upon the material presented in this chapter by summarizing the critical thinking skills, concepts, and terminology from the logic course.

Exercises

1. Analyze the following argument (identify the premises and the conclusion). Notice how the structure of the argument can be seen, even if the truthfulness of the premises is unknown or even if the language is unfamiliar.

> *A body on which a freely swinging pendulum of fixed length has periods of oscillation which decrease slightly with increasing latitude from the equator to both poles is an oblate spheroid slightly flattened at the poles.*
>
> *But the earth is a body on which a freely swinging pendulum of fixed length has periods of oscillation which decrease slightly with increasing latitude from the equator to both poles.*
>
> *Therefore the earth is an oblate spheroid slightly flattened at the poles.*

(W. A. Wallace, Einstein, Galileo, and Aquinas: Three Views of Scientific Method)

2. Analyze the following argument (identify the premises and the conclusion).

> *In spite of the popularity of the finite world picture, however, it is open to a devastating objection. In being finite the world must have a limiting boundary, such as Aristotle's outermost sphere. That is impossible, because a boundary can only separate one part of space from another. This objection was put forward by the Greeks, reappeared in the scientific skepticism of the early Renaissance and probably occurs to any schoolchild who thinks about it today. If one accepts the objection, one must conclude that the universe is infinite.*

(J. J. Callahan, "The Curvature of Space in a Finite Universe," Scientific American, August 1976)

3. Analyze the following argument (identify the premises and the conclusion). What kind of an argument is this? This argument may have been convincing in the 19th century, but is it convincing today? Explain. Hint: Are there really inhabitants on mars?

> *The planet Mars possesses an atmosphere, with clouds and mist closely resembling our own; It has seas distinguished from the land by a greenish color, and polar regions covered with snow. The red colour of our sunrises and sunsets. So much is similar in the surface of mars and the surface of the Earth that we readily argue there must be inhabitants there as here.*
>
> (W. Stanley Jevons, 19th century logician)

4. Explain why the following argument is invalid. The argument is presented in its original form and in standard form.

> *Total pacificism might be a good principle if everyone were to follow it. But not everyone is, so it isn't.*
>
> (Gilbert Harmon, The Nature of Morality)

[antecedent] [consequent]

P1 If everyone were to follow it, then pacificism might be a good principle.
P2 Not everyone is following it. (deny antecedent)
C Pacificism is not a good principle. (deny consequent)

5. Does the following argument lead to the truthfulness of the conclusion? Explain.

P1 If the sun is shining, then the house becomes warm.
P2 The house becomes warm.
C The sun is shining.

6. Find three short science articles (or paragraphs from longer articles) in newspapers or magazines that are arguments or explanations (see Appendix C) and write them in standard form. Please hand in a copy of the article with the analysis of the article.

7. Choose one of the characteristics of a good learner in Appendix B and write a conditional argument that has the following conclusion:

"Studying this book has helped me become a better learner"

8. Identify a scientific "fact" and write an argument to support the position that the "fact" is indeed true. Please refer to the Rutherford scattering experiment, which eventually resulted in the conclusion (or "fact") that protons exist, as an example of what is wanted in this problem.

III. Basic Mathematical Reasoning

A. Arithmetic and Basic Algebra

Basic mathematical reasoning is essential to everyone's life. It is used to determine where to buy gas, which shirt to buy, how much to tip the waitress, etc. Even though everyone has learned the basic mathematical reasoning skills needed to do basic science, it might be a good idea to review some of them. This chapter will review some of the specifics needed to understand basic science, but it is assumed that the rules of arithmetic are known and can be readily applied.

Some very basic arithmetic is required. As an example, consider the density of an object, which is a function of mass and volume. Being a function of mass and volume means that the density depends on mass and volume. Specifically the density is found by dividing the mass by the volume. If the letter d stands for density, the letter m stands for mass, and the letter V stands for volume, then the functional relationship could be written $d = m/V$. The mass, m, and the volume, V, are said to be variables of density. They can change, but knowing their values allows calculation of density, d.

$$\text{If } m = 2 \text{ and } V = 4, \text{ then } d = m/V = 2/4 = 0.5$$

When doing physical calculations it is important to express the units being used. Throughout this book units will always be used, even though they are not discussed in any detail. One common unit of mass is the kilogram which is abbreviated kg. A common unit for volume is the liter, abbreviated L. One common unit for density, therefore, is kg/L. The above calculation is more appropriately given as

$$\text{If } m = 2 \text{ kg and } V = 4 \text{ L, then } d = m/V = 2 \text{ kg}/4\text{L} = 0.5 \text{ kg/L}$$

The idea of using a symbol for a variable should already be familiar. The words we use to communicate with each other are just symbols (cow is a symbol for a four legged animal that produces milk). It should also be clear that the context of the symbol is important and that the same symbol could mean two different things (turn right, you are right). An example of that in science is that g could mean grams (a unit of mass) or it could mean the acceleration due to gravity (a number, usually about 9.8 meters per second squared or 9.8 m/s^2). The context will tell. If an object has a mass of 2.4 g, you would assume it means 2.4 grams. If, on the other hand, the symbol g is found in a mathematical equation, like $PE = mgh$ (potential energy equals mass times g times height), you would assume it to be the constant 9.8 m/s^2.

So far this example has only dealt with arithmetic and calculating density given mass and volume. Sometimes density and volume might be known and it is necessary to calculate mass. This is possible by rearranging the formula $d = m/V$ as $m = d*V$. There are several ways to explain how this rearrangement is done, but it is expected that one of them is already familiar to the reader. The relationship to find volume would be $V = m/d$.

B. Reporting Numbers

Any time a number is reported there should be some measure of how "accurate" it is. There should be an indication about how confident we are that the number is correct. A common practice today is for pollsters to provide a "margin of error" along with those numbers. The margin of error is reporting how confident they are that the result is meaningful. If a report says that 52% of the American people agree on an issue, but that it has a margin of error of 4%, it would suggest that the nation was pretty evenly divided. The people who reported the data think that the actual number could be anywhere from 56% down to 48%. If the margin of error was 10%, we would have less confidence in the reported number. The greater the margin of error, the less we believe the number.

In science every number should have an indication of the uncertainty in that number. Every number should have something like a margin of error. There are many ways to determine the uncertainty in a number. Scientists use instruments to get numbers. Each instrument has some uncertainty associated with it. A thermometer may have lines representing 0.5 ^{o}C between each line. You could tell that a temperature was between 21.0 and 21.5, but you would have to guess if it was 21.2, 21.3, etc. The uncertainty would be in the first decimal place. You might think that the reading should be 21.3 and that you are confident it is between 21.1 and 21.5. You could then report the number as 21.3 ± 0.2. The "plus or minus" would tell anyone who saw the number how much confidence you have in the number. In this case it says that the actual temperature is likely between $21.3 + 0.2 = 21.5$ and $21.3 – 0.2 = 21.1$. If you had a better thermometer, you could report more decimal places and the "plus or minus" would be smaller.

There are several mathematical ways to get the uncertainty. You may have heard of the standard deviation. It could be used to get an uncertainty number (a "plus or minus" number). A maximum differential error analysis could be performed to get the number for the uncertainty. In this class we will use a simple way of getting an uncertainty number using what is called the range for a set of data.

Consider the following set of data to illustrate the language and process of reporting numbers to be used in this class:

6.14, 6.03, 5.96, 6.11, 5.93

The sum of the numbers is 30.17 and the average is 6.034 (the sum divided by the number of data points). The median of a set of data is the data point that has exactly the same number of data points greater than itself as it has data points less than itself. In our case the median would be 6.03 since there are two numbers greater than 6.03 and two numbers less than 6.03.

The range is defined as the largest number minus the smallest number. In our case it would be $6.14 – 5.93 = 0.21$. The range is how far it is between the high and the low.

If the median and the mean are close to the same they will be in the middle of the range. So, if half of the range is added to the average the high number in the range would be obtained and if you subtract half of the range from the average the low number in the range would be obtained. In our case they aren't exactly the same, but they are close so that half of the range, 0.21/2 =

0.105, could be used as the “plus or minus”. For purposes of this class always assume the average and the median are close to the same.

It would seem that we should report 6.034 ± 0.105 as our result, but there is one more item to attend to. Notice that the "plus or minus" causes changes in the first decimal place. It is reported to the third decimal place, but does that five on the end matter if it changes in the first decimal place? The answer is no, the numbers beyond the first decimal place don’t matter. Both 0.105 and 0.12 could be rounded to 0.1. This means that the "plus or minus" should always be rounded to one non-zero number. In addition it should always be rounded up to make sure that all of the uncertainty is covered. In this case it should be rounded from 0.105 to 0.2. Rounding to 0.11 would not be right because the one in the hundredths place doesn’t make any difference when there are changes in the tenths place (0.11 is normally rounded to 0.1, but for an uncertainty it is rounded to 0.2).

Now that the "plus or minus" number is rounded off to the first decimal place, the number that is being reported for the result should also be rounded off to match it. The three and the four, in 6.034 of the number we are reporting, don’t matter if it is changing in the tenths place. Changing in the tenths place automatically changes the hundredths and thousandths places, so we should round the result, in the normal way, to the same decimal place as the "plus or minus", in this case to the tenths place.

The number to report would then be 6.0 ± .2, where both the reported value and the "plus or minus" number are given to the first decimal place.

The process is to calculate the average number and the "plus or minus" value, round the "plus or minus" value up, and then use the normal rounding rules to round the number being reported (the average) to the same decimal place as the "plus or minus" value.

Here are a few more examples.

$$4.381 \pm 0.062 \rightarrow 4.38 \pm 0.07$$
$$0.682 \pm 0.244 \rightarrow 0.7 \pm 0.3$$
$$0.1388 \pm 0.0038 \rightarrow 0.139 \pm 0.004$$
$$5.377 \pm 1.235 \rightarrow 5 \pm 2$$
$$2.867 \pm 1.3 \rightarrow 3 \pm 2$$

C. Mathematical Modeling

In science the goal is to understand and describe how nature works. The theories that are developed are the clearest, most concise way to describe nature. Scientific theories often use a combination of language and mathematics. It is important that students of science have some basic mathematical skills. This section will specifically address mathematical modeling using graphs, tables, and equations.

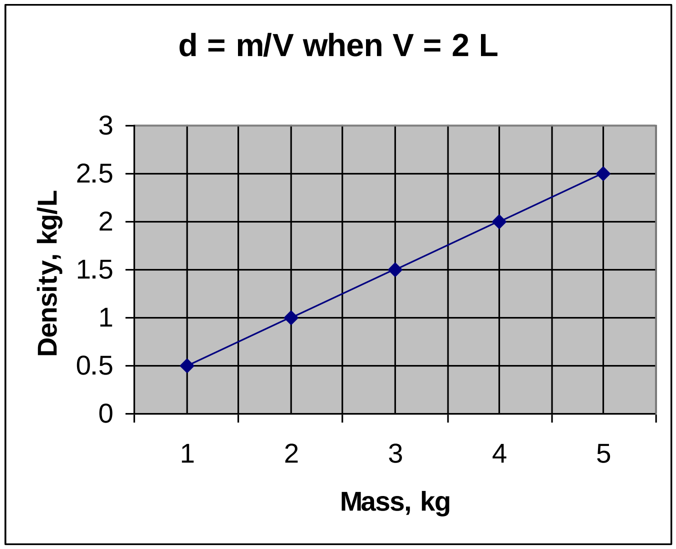
Graphing Basics

Graphs are separated into an x-axis, which is usually horizontal, and a y-axis, which is usually vertical. These axes are perpendicular, or at 90 degrees, with respect to each other. The range of values appropriate for the variable that is placed along the x-axis is evenly distributed along that axis. The same is done for the variable that is along the y-axis.

For a given value of the variable along x, the value of the variable along y is calculated from a mathematical relationship. Consider the density relationship, $d = m/V$, when $V = 2$ L. If the mass is 1 kg, the density will be 0.5 kg/L. If the mass is 2 kg, the density will be 1.0 kg/L. And so on. The following table shows the calculated values of density (when $V = 2$ L) for a few different masses.

Mass in kg	Density in kg/L
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5

If mass is placed along the x-axis and density is placed along the y-axis, then a graph like the one shown below can be generated by going along x to the value of one of the mass readings, and then going up along the y-axis to the corresponding y-value and putting a dot. This same process is repeated for each value of the mass. Reading a graph uses the same process in reverse. For a density of two, a horizontal line going through the value of two should be drawn across until it crosses the curve. A vertical line is then drawn downward through that intersection point until it crosses the x-axis. The value at that crossing point on the x-axis (4 kg) is the value of the mass corresponding to a density of two and a volume of 2 L.

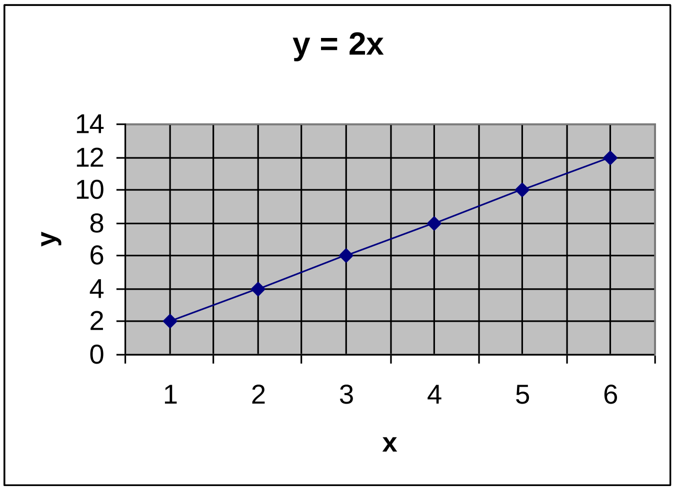


More Connections

To better understand the connections between graphs, tables, and equations consider the mathematical concept of directly proportional. A short statement that gets to this idea is if two things are directly proportional, then increasing one will also increase the other. The statement "y is directly proportional to x" could be written: y α x. To convert from a proportionality to an equation there needs to be a proportionality constant. These proportionality constants in science are obtained by experimentation. For this example assume the proportionality constant is two. The equation would then be $y = 2x$. A table can be constructed from the equation by picking an x and then calculating the corresponding y. Here is such a table.

x	y
1	2
2	4
3	6
4	8
5	10
6	12

These numbers can then be placed on a graph.



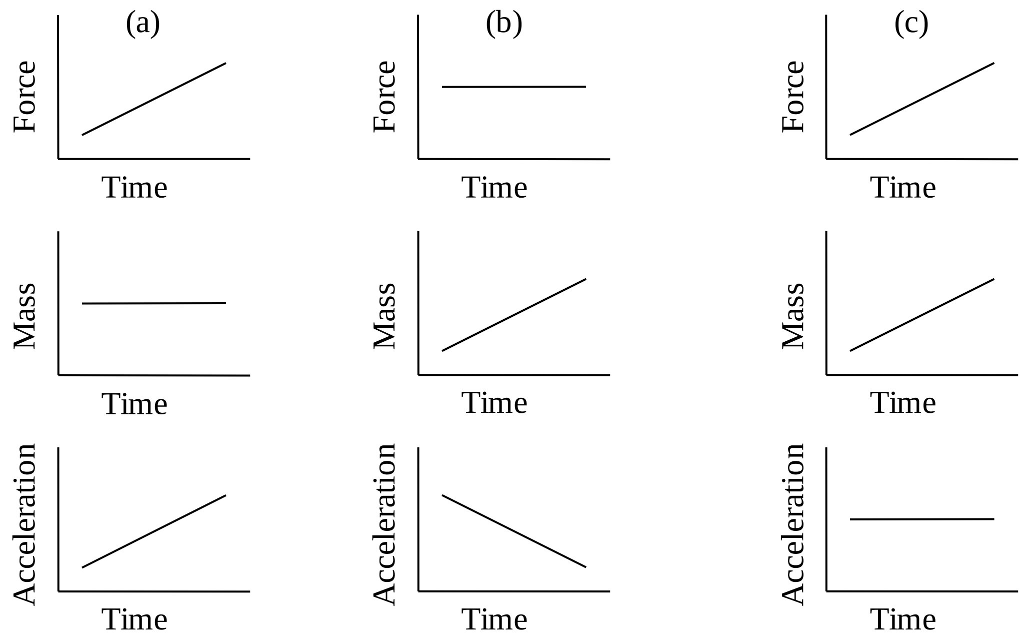
Notice that the equation, the table, and the graph are three equivalent ways of expressing the same thing. If someone gives you the equation, you could generate the table and the graph. If someone gives you the table, you could generate the equation, and the graph. If someone gives you the graph, you could generate the table and the equation.

Another mathematical concept that we will need is inversely proportional (or indirectly proportional). Generally speaking this says that if one variable goes up the other one goes down. There is a specific mathematical relationship, however, for inversely proportional. The equation for y inversely proportional to x is in the form: $y = 1/x$. That is, y is equal to one divided by x.

Application: Acceleration

Some physical properties, like density, have two variables associated with them. As another example consider acceleration. Acceleration has to do with the change in an object's motion. For an object to change its motion there must be an unbalanced force. The ability of a force to move an object depends on the mass, and so the acceleration depends on both force and mass. The greater the unbalanced force the more the motion will change and the greater the acceleration. In other words, the acceleration is directly proportional to the force. When the mass goes up the change in motion, or acceleration, goes down (it is harder to move a larger mass). In other words, the acceleration is inversely proportional to the mass. This is the language model. The corresponding equation would be $a = F/m$, where a is acceleration, F is the force, and m is the mass. Acceleration is the force divided by the mass.

Consider the following situations over a specified period of time: (a) the force is increasing, but the mass is constant, (b) the force is constant, but the mass is increasing, (c) both force and mass are increasing. How would the acceleration change in each case? Here are some graphs for each case.



Note that these are sketches and are intended to show the basic effect not the exact curve or slope of the line. The idea is that the force makes the acceleration go up when the mass is constant, the mass makes the acceleration go down when the force is constant, and when both force and mass increase at the same rate, the acceleration would not change. If the force increased faster than the mass, the acceleration would go up, but not as steeply as the force. When one goes up and the other goes down, the acceleration would depend on which one is changing faster.

Exercises

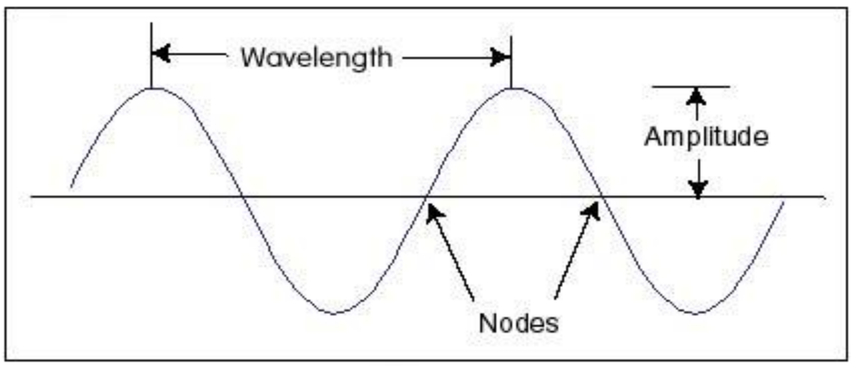
1. Given d = m/V, (a) calculate the density when m = 3 kg and V = 2 L.
   Answer: d = 1.5 kg/L.
(b) calculate the mass when d = 6 kg/L and V = 3 L.
   Answer: m = 18 kg
(c) calculate the volume when m = 5 kg and d = 4 kg/L.
   Answer: V = 1.25 L.
2. If $E_p = mgh$, calculate h when (a) $E_p = 100$, m = 5, g = 10. (b) $E_p = 75$, m = 4, g = 10.

3. (a) Predict the density vs time graph, like in the examples above, for an experiment that determines density by changing mass at a constant volume. (b) Predict the density vs time graph for an experiment that determines density by changing volume given a constant mass.

IV. Light and Atomic Theory

A. Waves

A standing wave has disturbances at regular fixed intervals. The wavelength, λ, has units of meters and can be measured from peak to peak, valley to valley, or between any two points that are identical. The frequency, υ, is the time for one cycle to occur (that is the time for one wavelength to go by). The unit of frequency is cycles per second or Hertz, Hz. The amplitude, A, (maximum height of the wave) and nodes (zero amplitude) are illustrated below.



For a transverse wave, the disturbance is at right angles to the propagation of the disturbance. Light is often thought of as if it was made of transverse waves. This explains many of the properties of light. Light, or any wave, can be equivalently described by wavelength, frequency, energy, and color (at least for visible light). Energy is inversely proportional to wavelength and directly proportional to frequency. Blue has a shorter wavelength than red and, therefore, a higher energy and higher frequency. Visible light is only a small part of the electromagnetic spectrum. Infrared light, which we can't see, is responsible for the warmth we feel when standing in the sunlight. Ultraviolet light can't be seen either, but it is responsible for sunburns. Radio waves, microwaves, x-rays, and many others are also different kinds of "light". Each has a characteristic wavelength, frequency, and energy.

B. Elements and Atomic Structure

Elements are the basic building blocks of all matter. There have been at least 111 elements identified. Everything on earth is made from these elements. Atoms are the smallest piece of an element that retains the characteristics of that element. Atoms are composed of protons, neutrons and electrons. The protons and neutrons are almost 2000 times larger than the electrons and reside in the massive nucleus. The electrons are outside of the nucleus, but do NOT orbit around the nucleus like planets around the sun. If the nucleus of a hydrogen atom was the size of the earth, the electron would be half way to the sun. There is a lot of space in the atom.

Each element has a distinctive kind of atom. The number of protons in the nucleus determines which element it is. The periodic table allows one to easily determine how many protons are in the atoms of each element by looking at the number that runs consecutively across the table. That number not only numbers the elements, but also designates how many protons there are in the atom of that element. Neutral atoms have the same number of electrons as protons, since the electron has a charge of -1 and the proton has a charge of +1. Note that the number of electrons in an atom can change and that the number of electrons does not identify the element (unless the atom is neutral).

The atomic theory described above has been developed over many years. At one point in history the notion of matter being composed of discrete particles was a radical idea. But more and more evidence came forward to substantiate this view. One theory that was popular for a while was the raisin pudding model. According to this model the atom was composed of a spread-out positively charged cloud with electrons scattered throughout, like raisins (electrons) in pudding (positive cloud). This theory had to be discarded when Rutherford found that alpha particles (two protons and two neutrons) were scattered by gold atoms. Rutherford proposed a planetary model that was popular for several years, but it too was shown to be wrong through experimentation. The current model is probabilistic in nature and uses quantum mechanics to describe the atom. The current model can predict a region in space where an electron is likely to be for a given energy state, but it can’t determine the exact location of the electron.

The composition of the atom is a topic of current research. Physicists have created a theory that proposes even more fundamental particles than the proton and neutron. Over 200 subatomic particles have been identified. Protons and neutrons are now thought to be made from particles called quarks. These quarks have "color" and "flavor". This is a strange new world, but very exciting.

C. Atomic Structure, Line Spectra, and Light Absorption

Quantum theory predicts that electrons reside in energy states and that the electrons can move to higher energy states when excited. They can be excited by light, heat, electrical energy, etc. Electrons, like everything else, will tend toward the lowest energy state. So, after they are excited by the light or other energy, they "fall" back down to the lower energy state. Conservation of energy requires that energy be given off when going from a high energy to a low energy. This energy is often given off as light.

A curious finding from observing the light given off as electrons fall back to lower energy states is that each element gives off characteristic energies of light. It is also found that each element only absorbs certain characteristic energies. Furthermore, the energies absorbed and the energies emitted are the same! This leads to the conclusion that the energy levels are discrete, like steps on a ladder. The electrons can't be found between steps. The energies observed form a line spectrum. It is distinguished from a continuous spectrum (which has all possible colors) in that only certain colors are observed, with dark spaces in between.

The selective absorption of light by atoms and molecules gives rise to much of the color around us. The leaves are green because chlorophyll only absorbs the complement of green (when the complement of green is absorbed we see green). Neon signs, night sticks, glow in the dark toys, and fireworks give off their characteristic colors due to this quantum effect. The world would certainly be a boring place without quantum effects!

V. Intermolecular Forces

A. Charge, Elements, and Compounds

Electrons are negatively charged and protons are positively charged. Since electrons are about 2000 times smaller than protons, atoms become charged by losing or gaining electrons. If an atom gains an electron, it will have a negative charge. If an atom loses an electron it will end up with a positive charge. Charged particles are called ions.

Some elements prefer to lose electrons. Sodium (Na), for instance, readily gives up one electron to form a positive ion, designated Na^+. All of the elements in the first column of the periodic table will easily give up one electron to form a positive ion with a charge of 1+. Each column of the periodic table designates a family of elements that act alike. All of the elements in the second column (or family) of the periodic table will readily lose two electrons and become positive ions with a charge of 2+. Barium, Ba, forms the Ba^{2+} ion, for instance.

Other elements prefer to gain electrons. The next to last family of elements (F, Cl, Br, etc.) are called halogens and like to gain one electron, forming a negative ion with a charge of 1- ("one minus" or "minus one"). F^-, Cl^-, and Br^- are examples of these ions. Oxygen prefers to gain two electrons to form the O^{2-} ion.

For one atom to gain an electron, another atom must give up an electron. This happens simultaneously, forming a positive and a negative ion. Since ions with opposite charges attract, it is possible to form compounds where different elements are bound together due to the attraction between charges. Common table salt, sodium chloride or NaCl, is an example. The sodium allows an electron to be transferred to the chlorine to form Na^+ and Cl^- ions that are attracted to each other and form table salt, Na^+Cl^-.

If an atom wants to give up two electrons, each electron must have a place to go. This keeps all compounds neutral. A barium atom would need two chlorine atoms in order for all of the electrons to be accounted for. Barium chloride would have to be $BaCl_2$. Since oxygen receives two electrons, BaO would be an acceptable compound. All compounds formed by the attraction of negative and positive ions are called salts.

Just as atoms can form ions, groups of atoms that are connected to each other by sharing electrons can form ions. Some examples are OH^-, NO_3^-, NH_4^+, SO_4^{2-}, HCO_3^-. These ions are also formed by other elements, or groups of atoms, giving up or gaining the appropriate number of electrons. Some compounds are Na^+OH^- (the major component of Drano), $NH_4^+OH^-$ (ammonia in water, sometimes used to clean windows), $Na^+HCO_3^-$ (baking soda). These polyatomic ions do not easily come apart and remain together under normal circumstances. Putting baking soda into water, for instance, causes the Na^+ and the HCO_3^- to separate while in solution, but the HCO_3^- remains as one unit.

Acids are formed when the positive hydrogen ion, H^+, is paired with a negative ion. HCl is hydrochloric acid, HNO_3 is nitric acid, H_2SO_4 is sulfuric acid, etc. The odd thing about acids is that they stay together by sharing electrons, but they act like ionic compounds by separating into ions when placed in water. Bases are formed when a positive ion is paired with the hydroxide ion, OH^-. NaOH is sodium hydroxide, $Mg(OH)_2$ is magnesium hydroxide, $Al(OH)_3$ is aluminum hydroxide, etc.

When an acid is mixed with a base, water and a salt are formed. HCl mixed with NaOH will give water, HOH or H_2O, and table salt, NaCl. Acid base indicators are generally materials that have one color in an acid solution and a different color in a base solution.

B. Nonpolar and Polar Molecules

Molecules are formed when atoms are bound together due to them sharing electrons. If the atoms are identical (e.g. O_2) they have equal ability to attract electrons. When this is the case the molecules are said to be nonpolar. If a molecule is completely symmetrical it is nonpolar.

Many molecules have different elements combined together (e.g. CO). When this happens there is a "tug of war" for electrons that takes place between the atoms in the molecule. Since some elements like to gain electrons, while others like to give them up, the electrons will spend more time on the side of the molecule where the atom with the strongest ability to attract electrons resides. In the example of carbon monoxide (CO) the oxygen has a stronger ability to attract electrons and so the oxygen end of the molecule has a small negative charge. The charge is less than one, since the electron is not actually transferred. The other end of the molecule (the carbon end) is somewhat electron deficient and therefore has a small positive charge (the same magnitude as the negative charge on the other end). When this kind of situation exists the molecule has a dipole and the molecule is said to be polar.

C. Molecular Interactions

Because polar molecules have positive and negative ends, they are able to interact with each other. If the molecules are able to freely orient themselves, they will rotate so that the negative end of one molecule is near the positive end of another molecule (unlike charges attract). This attraction is a weak bond between molecules and is one kind of intermolecular interaction. These interactions are responsible for a number of things that we commonly observe, such as the separation of oil and water, the tendency of water to form spheres, and the cleaning ability of detergents.

Exercises

1. Why is a Rice Krispie attracted to a charged balloon? Why does it fly away after touching?

2. Why is water attracted to a charged rod?

3. How does detergent remove oil from your hands?

VI. Basic Physics

A. Kinetic Energy, Potential Energy, and Conservation of Energy

Units: The unit of kinetic and potential energy is the Joule, J. The calorie is another common unit of energy. The unit of mass is the kilogram, kg (or gram, g). The unit of distance is meter, m (one meter is about one yard). The unit of time is second, s. The unit of speed is meter/second, m/s. The unit of acceleration is meter per second squared, m/s^2.

Kinetic energy is the energy of motion. Every object that is moving has kinetic energy greater than zero. Kinetic energy depends on the mass of the object and how fast the object is moving. Potential energy is due to position or structure. Gravitational potential energy is due to gravity and depends on the mass and the height above the ground (or floor, or table, etc.) of the object. The place to measure from (ground, floor, table, etc.) is determined by the physical situation and is generally taken as the lowest place that the object could go. Only gravitational potential energy will be considered in this book.

The mathematical model for kinetic energy is given by one-half times the mass times the square of the velocity,

$$KE = \tfrac{1}{2}mv^2.$$

If a person with a mass of 50 kg (about 125 pounds on earth) is traveling at a speed of 1.44×10^8 m/s (40 km/hr or about 25 miles per hour) they would have kinetic energy equal to:

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(50\text{ kg})(1.44 \times 10^8\text{ m/s})^2 = 36 \times 10^8\text{ J}$$

The mathematical model for gravitational potential energy is given by mass times the acceleration due to gravity times the height,

$$PE = mgh,$$

where g is the acceleration due to gravity or 9.8 m/s^2. A 50 kg person that was 3 m above the ground would have a potential energy equal to:

$$PE = mgh = (50\text{ kg})(9.8\text{ m/s}^2)(3\text{ m}) = 147\text{ J}$$

Conservation laws are common in science. One statement of conservation of energy is that energy can not be created or destroyed, but can only change form. All conservation laws, in one way or another, say that what you start with is what you end up with. One mathematical model for conservation of energy is written as follows:

$$KE_i + PE_i = KE_f + PE_f + W$$

In this model the total initial kinetic energy, KE_i, plus the total initial potential energy, PE_i, equals the total final kinetic energy, KE_f, plus the total final potential energy, PE_f plus any other energy that might have been formed (like heat from friction), W. All energy is accounted for, both before and after, and the energy before is found to equal the energy after.

B. Momentum and Conservation of Momentum

Momentum is a measure of the tendency of an object to remain in motion, if in motion, or at rest, if at rest. Momentum is proportional to both mass (it is harder to move a large mass) and velocity (it is harder to deflect a fast moving object). Momentum, p, is calculated by multiplying mass, m, times velocity, v.

$$p = mv$$

Momentum, like energy, is conserved. The total initial momentum must equal the total final momentum. For two objects, A and B, that collide, the mathematical model would be:

$$p_{Ai} + p_{Bi} = p_{Af} + p_{Bf}$$

Initial would be before collision and final would be after collision (indicated by the subscripts i and f, respectively). The conservation equation could also be written:

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

where mass times velocity has been substituted for momentum (p = mv).

Conservation of momentum is very important when trying to understand many common occurrences. Rockets are propelled due to conservation of momentum. Consider a rocket at rest in space. While at rest the momentum is zero (it isn't moving and so the velocity is zero and any mass times zero is still zero). When the fuel is expelled out of the back of the rocket, it (the fuel) has some momentum (the fuel has mass and velocity). But the initial momentum, which was zero, must equal the final momentum. The rocket must move in a direction opposite to the fuel (making the rocket's momentum negative relative to the fuel's momentum) in order for the rocket's momentum to cancel out the momentum of the fuel. The final momentum of the fuel plus the final momentum of the rocket must equal zero (because the initial total momentum was zero).

Conservation of angular momentum (momentum of objects traveling in a circle) is responsible for the way ice dancers can do the amazing spins that are generally seen at the end of their performances. It is also responsible for the design of helicopters, the ease of riding a bicycle, the way a gyroscope works and the way guidance systems of many airplanes and other aircraft work. Conservation of momentum is important!

C. Two Dimensional Physics

An interesting observation is that the horizontal axis is independent of the vertical axis. Velocity in the northeast direction may just as well be described as the combination of two velocities, one toward the north and another toward the east. The combination is equivalent to the original. This allows one to calculate the force that must exist when two forces going in different (but not necessarily opposite) directions are in equilibrium with another third force.

Another result of this horizontal and vertical independence is that a horizontal force will not change the vertical motion of an object. A ball falling straight down from rest will hit the floor at the same time as a ball that begins at the same height, but is also given a push in the horizontal direction. That seems somewhat counter-intuitive, but is nevertheless true. This is important in trajectory studies, propelling rockets, and any other situation when an object is propelled.

D. Forces and Equilibrium

Units: The unit of force is the Newton, N.

The concept of force is familiar to most people. There are economic forces, social forces, physical forces, governmental forces, etc. When any of these forces are not balanced, motion occurs. Unbalanced social and economic forces may cause motion in the form of riots, unbalanced governmental forces may give one person too much authority and "move" the country toward a dictatorship. Unbalanced physical forces also cause motion. If a ball is pushed, it rolls. The push is the force. If a ball is pushed equally from opposite directions, however, it remains stationary, independent of the amount of force applied. When forces are balanced they are said to be in equilibrium. If a light fixture hangs from the ceiling it is in equilibrium. If someone holds on to the light fixture causing the force of gravity to be greater than the force holding the person and light fixture to the ceiling, then motion will occur. They will fall to the floor!

When an object is not moving it is said to be in a state of static equilibrium. When static equilibrium exists, all of the forces, in all directions, must balance. If a ball is stationary on a floor, then all horizontal forces must cancel. An extra force, like wind, could cause motion, but while there is no motion horizontally the horizontal forces must all cancel. The same is true in the vertical direction. The force of gravity (which is always exerted toward the center of the earth) must be exactly equal to the force of the floor pushing up on the ball. If gravity pulled harder, the ball would go through the floor!

This idea of balance allows one to predict forces when equilibrium exists. If, for instance, there is a force of 10 N horizontally toward the north on an object and the object isn't moving, then there must be a force (or combination of forces) of 10 N applied horizontally toward the south. Any other situation would cause the object to move.

Exercises

1. If a 2.5 kg ball is dropped from rest at a height of 2.0 m, how much kinetic energy does the ball have when at a height of (a) 1.5 m, (b) 1.0 m, (c) 0.5 m, (d) 0.0 m? Assume no heat or other energy is produced.

2. What is the velocity of the ball at each height in problem one?

3. If 5 J of heat is produced during every 0.5 m that the ball travels, what will the kinetic energy of the ball be at each of the heights given in problem one?

4. How much heat is generated if a 3.0 kg ball dropped from rest falls 4.0 m and has 100 J of kinetic energy after falling that 4 m?

5. Explain how a rocket can change directions in outer space.

6. If a person stands on a platform that is able to rotate without friction, points a cordless drill toward the sky, and then turns it on, what will happen? Explain.

Appendix A

MODELING AND SCIENCE

One quality of a scientist is the ability to ask and logically answer the following questions about an observed situation:

What is happening?
Why is it happening?
What does it mean?

These three questions could be broken down as follows:

What is happening?
- Observation (statement of perceived facts)
- Description of the situation
- Identification of important aspects (variables)

Why is it happening?
- Modeling (interpretation of facts, theories)
- Language model (including both written and oral)
- Mathematical model (including graphs)
- Physical model (including sketches)
- Matter rearrangement model (chemical equations)

What does it mean?
- Prediction (applying a model to a new situation)
- Sensitivity analysis
- Limit analysis

Modeling is central to a scientist's activities. It might be said that a scientist's goal is to correctly model observable events. The models can take several forms. Here are a few:

1. Language models, both written and oral. These models use words to clarify observed events. Some people may consider a description of what happened to be a sufficient model, but for a scientist the description is only the framework of a good model. Often these models are called theories. These models often explain, in a logical way, why something happens. They may also show interconnections between variables and can be used to predict other events.
An example: The weight of an object depends on the mass of the object and the attraction of the object by gravity.

2. Mathematical models. These models use the language of mathematics to clarify observed events. Mathematics is a very powerful tool to simply define the connections between variables for some observed event. Mathematical models may include every variable in the system (if the model builder knows all of the variables), but is more likely to only include the most important variables. The value of the model has to do with its ability to correctly describe the observed

event. Each variable retained in the model is given a symbol and the symbols are related by a mathematical equation.

The idea of using a symbol for a variable should be familiar since the words we use to communicate with every day are just symbols (cow is a symbol for a four legged animal that produces milk). It should also be clear that the context of the symbol is important and that the same symbol could mean two different things (turn right, you are right). An example of that in science is that g could mean grams (a unit of mass) or it could mean the acceleration due to gravity (a number, usually about 9.8 meters per seconds squared). The context will tell. If an object has a mass of 2.4 g, you would assume it means 2.4 grams. If, on the other hand, the symbol g is found in a mathematical equation, you would assume it to be the constant 9.8 m/s^2. The mathematical model corresponding to the language model above is: weight = mass times the gravitational constant, or, using shorter symbols, wt = mg.

3. Graphical models. Graphical models are generated by choosing one variable to observe and then allowing a different variable to change over a range of values. For the model being used, as an example, you may wish to observe weight and see how it changes as the mass goes from zero to 100. Mathematical models and graphical models have a direct relation, knowing one determines the other. You can, however, predict graphical models from language models just as well when words like directly proportional, inversely proportional, etc. are used to describe relationships between variables.

4. Physical models (and their corresponding paper and pencil sketches). These are actual, physical systems that mimic the observed event. Some successful physical models have nothing to do with the observed event (frying an egg to model your brain on drugs), but most of the time they are systems that retain the important variables, but aren't too cluttered by incorporating all of them.

5. In chemistry and nuclear physics there are what I call matter models. These are symbolic relationships, similar to mathematical models (equations), which describe the rearrangement of matter. They are often called chemical or nuclear equations. An example is the combination of oxygen and hydrogen to form water.

$$2H_2 + O_2 \rightarrow 2H_2O$$

In the end models help us understand physical situations. If a model agrees with our observations, whether it makes any common sense or not, it is a good model. Good models agree with observations.

Appendix B

"MODEL OF A GOOD LEARNER"
by
Daniel K. Apple

A GOOD LEARNER:

1. Uses their **self esteem, confidence, and self worth** to tackle the unknown with knowledge that he/she will succeed in mastering each new learning exercise. Each new successful challenge increases your abilities to learn quicker and solve the next problem.

2. Demonstrates the **interest, motivation, and desire** to seek out new information, concepts, and challenges so he/she can apply them to new and exciting problems.

3. Uses **inquiry, questioning, and critical thinking** to be more efficient with time and to gain more insights in how concepts can be applied.

4. Engages each of his/her senses to access information, including **observing, touching, tasting, smelling, and hearing; with special emphasis on listening and reading.**

5. **Clarifies, validates, and assesses his/her own understanding of a concept through verbal and written presentations**.

6. Integrates each new concept within a general **systems perspective** and quickly **grasps instructions in a logical structure**.

7. **Accesses information** quickly and filters relevant data from irrelevant information.

8. **Experiments, discovers** and is secure in his/her emotions, so he/she can **risk** his/her **security**, and **accept failure** as a frequent occurrence in the course of succeeding at a new task.

9. **Develops stronger learning skills** and processes to support their use of the **Learning Process Model**.

10. Demonstrates strong **social skills, easily interacts with other people,** and is enjoyable member of any productive team.

11. **Invests in learning new tools**, especially computers.

12. **Focuses and concentrates** his/her energy on the important task at hand.

13. Develops an understanding of his/her **value system** and an **appreciation for other people's value system** and applies this value system in his learning experiences.

14.Clarifies his/her **life's goals and objectives** to apply his/her energy and accomplishes measurable outcomes within his/her strategy.

15. **Visualizes, models, transfers and synthesizes concepts**.

Appendix C

CRITICAL THINKING SKILLS, CONCEPTS, AND TERMINOLOGY

FROM THE BASIC LOGIC COURSE

by

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With reference to and examples from the text **INFORMAL LOGIC: POSSIBLE WORLDS AND IMAGINATION** by John Eric Nolt, McGraw-Hill, 1984, ISBN 0-07-046861-3

Logic does not deal with every form of thinking, language and expression. Logic deals only with argumentative thinking and expression.

In this context, "argument" should not be taken to mean dispute, disagreement, or fight. Rather, as used in logic, "argument" refers to a series of statements, one of which, called the **conclusion**, is intended to be evidentially supported by the other(s), called the **premise(s)**. (Nolt, p. 2)

That is, whenever available evidence or known information, expressed as a statement(s), is used to prove the truth of another statement, then we can say we have engaged in argument. The evidence or known information constitutes the premise material in the argument. The statement whose truth is intended to be proved or demonstrated is the conclusion.

The purpose of an argument is to prove or demonstrate the conclusion; to convince one's audience (oneself or others) of the truth of the conclusion statement.

Some arguments are simple inferences, in which there is only one conclusion, supported by one or more premises.

e.g. Hemlock is poison, so you shouldn't drink it.

P1 Hemlock is poison.
C You shouldn't drink it.

All even numbers are divisible by two. And all prime numbers are divisible only by themselves and one. Therefore, two is the only even prime number.

P1 All even numbers are divisible by two.

P2 All prime numbers are divisible only by themselves and one.

C Two is the only even prime.

Some arguments are complex, made up of two or more interconnected inferences, in which there is only one overall or final conclusion, but in which there is at least one intermediate conclusion. In the case of a complex argument, statements that function as premises, and are not supported by any other premises, are called basic premises (BP) or assumptions. These represent the common ground known or agreed upon by author and audience and from which the rest of the argument will flow. Statements that function both as the conclusion of one inference and as a premise for another inference within the same complex argument are called either non-basic premises (NBP) or intermediate conclusions (IC).

e.g. A computer is a machine, an artificial assembly of electronic and mechanical components. Therefore, all its operations are predictably determined by physical laws. But the operations of a mind cannot be predictably determined by physical laws. Consequently, computers do not have minds.

(1) BP ------------- A computer is a machine, an artificial assembly of electronic and mechanical components.

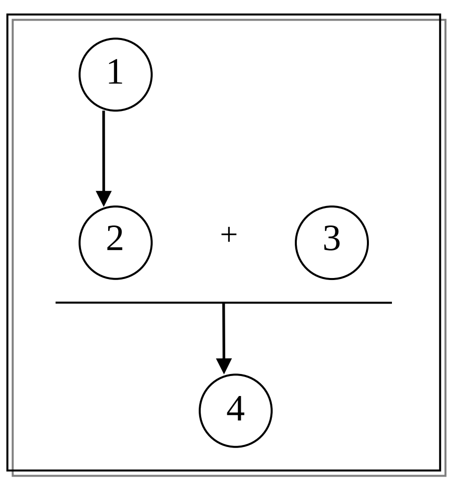
(2) IC/NBP ------- All its operations are predictably determined by physical laws.

(3) BP ------------- The operations of a mind cannot be predictably determined by physical laws.

(4) C --------------- Computers do not have minds.

In this example, statement #1 is offered as a premise to support statement #2 (see "therefore"). Statement #1 is not itself supported by any other statements. It is, therefore, functioning in the argument as a basic premise (BP) or assumption. Statement #2 is functioning as the conclusion of this first simple inference. It is not, however, the final conclusion of the complex argument as a whole. It is thus called an intermediate conclusion (IC). It is also called a non-basic premise (NBP), since it is also functioning as a premise, working with statement #3, to support statement #4, which is the main point and overall conclusion (C) of the argument. Statement #3 is not supported by any other statements, so, like statement #1, it too is a basic premise (BP).

A **diagram** of this argument can also be useful to display its argumentative structure. See below.



Arguments, of course, can be much longer and of any degree of complexity, but always consisting of basic premises, intermediate conclusions/non-basic premises, and a final conclusion similar to the argument above.

I want to emphasize here that nothing has been said up to this point about argument evaluation; whether the premises in the above arguments actually do support the conclusions, and, if so, how strongly. That issue will be addressed below.

For the purpose of recognizing arguments and the parts of arguments, it is useful to be aware of various commonly used linguistic clues that we give one another in ordinary language to signal the argumentative thrust of our communication. In the examples of arguments given above, the words "so," "and," "therefore," "but," and "consequently" are all functioning in that way to help us catch the argumentative drift in each case.

Some of these clues are called conclusion indicators because they are often prefixed to conclusion statements within arguments to signal that the forthcoming statement follows from and is intended to be supported by the preceding statements(s). "So," "Therefore," and "consequently" are common conclusion indicators.

Other linguistic clues are called premise indicators because they are often prefixed to premise statements within an argument to signal that these statements are offered to support some other statement, which other statement would be the conclusion of the argument. "For," "since," and "because" are common premise indicators.

Still other linguistic clues are called transition expressions. These expressions are not specifically premise or conclusion indicators, but they do often link premise statements with one another within an argument. In the examples above, "and " and "but" are transition expressions. Other common transition expressions are: "however," "moreover," "in the first place," " in the second place," "finally," etc.

It is important to realize that arguments can be quite successfully expressed without any of these linguistic clues; and the mere presence of any of these expressions does not by itself guarantee that an argument is present. However, these are extremely useful signals, commonly used in ordinary language, to help us recognize arguments and the parts of arguments. The following list is a more extensive sampling of such clues:

Conclusion Indicators:

Therefore	Accordingly	This entails that
Thus	Consequently	This proves that
Hence	This being so	From which we can deduce that
So	It follows that	As a result, we may infer that
Then	Which implies that	

Premise Indicators:

For	Granted that	Given the fact that
Since	Inasmuch as	As is implied by the fact that
Because	The reason is that	This is true because
Assuming that	In view of the fact that	Seeing that

Transition Expressions:

But	Still	First
And	Yet	Second
However	Although	In the first place
Moreover	Likewise	In the second place
Furthermore	Similarly	Finally
Nevertheless	In addition	Also

Some passages we meet in ordinary language may look like arguments, because they consist of a series of statements and may even employ some of the linguistic clues listed above, yet they are not arguments. It is thus important to distinguish arguments from descriptions and explanations.

Descriptions simply state facts, provide information, describe. No support of any kind is given. No statements are offered to support any other statement. Transitional expressions may often occur within simple descriptions, but neither premise nor conclusion indicators do so.

Explanations look even more like arguments because they use the same battery of linguistic clues that arguments use. However, their purpose and cognitive context are quite different. Whereas the purpose of an argument is to convince or prove to someone that such and such is the case, the purpose of an explanation is, already knowing that it is the case, to say why.

The following schematic presentation shows the similarity and parallelism of structure between arguments and explanations, but also the difference in initial cognitive context, i.e. what is known initially or at the out set of discussion and what part is being sought:

Argument Explanation

Premise(s) ----- known initially Explanans ----- not known initially,

to be discovered

Conclusion ----- not known initially, Explanandum - known initially

to be proved

e.g. "I went to the woods because I wished to live deliberately, to front only the essential facts of life, and see if I could not learn what it had to teach, and not, when I came to die, discover that I had not lived." (H. D. Thoreau, Walden)

Notice the indicator "because." That clue tells me that what comes after "because" is offering some kind of support for the opening statement. Thus, the passage could not be merely descriptive. It must be either an argument or an explanation. I then ask whether Thoreau in this passage is trying to prove that he went to the woods (in which case the opening statement would be the conclusion of an argument) or is he stating why he went to the woods (in which case the opening statement is the explanandum of an explanation.) The information after the "because" does not seem designed to prove that he went to the woods, but, already knowing that he went, would tell the reader why. The passage is better interpreted, then, as an explanation. The larger context within the book from which the passage comes would give the reader even more help in interpreting this passage.

". . . we know that there is no greatest prime number. But of all the prime numbers that we shall have ever thought of, there certainly is a greatest. Hence, there are prime numbers greater than any we shall have ever thought of". (Bertrand Russell, "On the Nature of Acquaintance")

The indicator "hence" shows that the last statement is the focal point of this passage, either a conclusion or explanandum. The transitional expression "but" links the first two statements, showing that they are working together and serving a similar function, in this case to support the third. And this is the case whether the passage turns out to be an argument or an explanation. In this case the passage itself and, by the way, the larger context from which it is derived, suggests that Russell is not trying to explain why there are such prime numbers greater than any we shall have ever thought of, but rather that he is trying to prove or convince the reader that there are such prime numbers. The passage is thus an argument. The first two statements, better known initially by the audience, are thus premises, the last statement, which is being proved, is the conclusion.

The worksheet at the end of this appendix provides further opportunities to test critical thinking skills in differentiating descriptions, explanations, and arguments. Answers with brief notes are also given. This is a worksheet used in the Logic course for that purpose.

One final point that should be mentioned here is the evaluation of arguments. In addition to recognizing and analyzing arguments, logic is also concerned with the process of evaluating arguments. A substantial portion of the Logic course is devoted to that part of the process.

As indicated above, the purpose of an argument is to prove, demonstrate, convince one's audience that the conclusion statement is true. For an argument to succeed in this purpose, two criteria must be satisfied:

(1) the reasoning or inferential link between premise(s) and conclusion must be sufficiently strong to make our acceptance of the conclusion statement reasonable in light of the stated premise(s);

(2) the premises, on the basis of which we draw the conclusion, must be true.

Under criterion #1 we identify three different strengths of reasoning: deductive (which is the strongest reasoning), inductive, and fallacious (which is the weakest reasoning).

A deductive argument is one in which if the premises were true, then the conclusion would have to be true.

e.g. P1 -- All bananas are purple.

P2 -- All purple things are vegetables.

C --- All bananas are vegetables.

This is a deductive argument. What makes it deductive is not that the premises are true. In fact, in this simple and rather silly example we can see that the premises are false. However, the inferential link between these two premises and this conclusion are such that if the premises were true, then the conclusion would have to be true. That is all that is required for the argument to be deductive. By the way, this argument, although deductive, will still not convince anyone that the conclusion is actually true, because the premises are not true. But that has to do with the second criterion above. This argument satisfies the first criterion. Its reasoning is strong, in fact, as strong as it can possibly be, i.e. deductive.

An inductive argument is an argument in which if the premises were true, then there would be a greater than 50% (but less than 100%) probability that the conclusion statement would be true.

e.g. P1 -- Nearly all early 20th century composers used considerable dissonance in their music.

P2 -- Charles Ives was an early 20th century composer.

C --- He must have used a considerable amount of dissonance in his music.

This is an inductive argument. The quantifying expression "nearly all" puts the argument into the range of very high probability, but not 100% necessity. Thus it can be said that, if these two premises were true, then there would be a very high probability (certainly greater than 50%) that the conclusion would also be true. That is the condition that must be met to consider the argument's strength of reasoning to be inductive.

A fallacious argument is an argument in which, even if the premises were true, there would be a less than 50% probability that the conclusion statement would also be true.

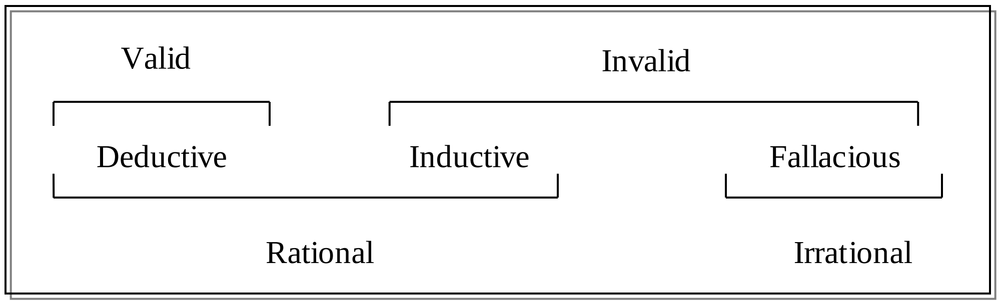
e.g. P1 -- A few fish are warm-blooded animals.

P2 -- The great white shark is a fish.

C --- The great white shark is warm-blooded.

The quantifying expression "few" is the tip-off expression here. The reasoning is fallacious, that is, too weak to reasonably accept this conclusion in light of these premises. The fact that both premises are actually true does not salvage the weak inferential link between premises and conclusion. This argument fails criterion #1.

Other logical terms also used for the strength of reasoning criterion are indicated by the following chart:



Criterion #1 is satisfied if the reasoning is deductive or at least inductive. If the reasoning is fallacious, it is simply not reasonable to accept the argument (that is why the chart identifies fallacious with irrational).

In the Logic course we develop two methods for evaluating the strength of reasoning of arguments: the syllogistic method and the possible worlds method.

If criterion #1 is satisfied, then we have to determine next if the argument also satisfies criterion #2, i.e. Are the premises true?

Only if both criteria are satisfied --- deductive or inductive reasoning <u>and</u> all true premises --- can the argument give assurance about the truth of the conclusion --- that it is necessarily true, that it is quite probably true, etc.

The following statements can be used to practice some of the ideas presented in this appendix. Determine if the following statements are descriptions, explanations, or arguments.

1. Mike is a burglar. He is very successful at it and he seldom has to do an honest day's work.

2. John is uncouth, he has very little interest in games, and, in fact, he much prefers to read.

3. Alice stopped taking him out because he was always late.

4. Kathy had a good high school education. Mike, Nancy, and Arthur also had a good high school education and they went to Central High. So Kathy must have gone to Central High too.

5. Since Julius Caesar conquered Gaul, there have been numerous geopolitical changes in Western Europe.

6. If this chewing gum contains sugar, then it's fattening. But if it's fattening, then you shouldn't buy any. Thus, if this chewing gum contains sugar, then you shouldn't buy any.

7. Several members of the Omega Club have substantial holdings in real estate. One member, Tracy Hawkins, owns over 100 condominiums.

8. If two equal molecules are formed of the same substance and have the same temperature, each receives from the other as much heat as it gives up. Their mutual action may thus be regarded as null, since the result of this action can bring about no change in the state of the molecules.

9. I believe that the testing of the student's achievements in order to see if he meets some criterion held by the teacher is directly contrary to the implications of therapy for significant learning.

10. No business concern wants to sell on credit to a customer who will prove unable or unwilling to pay his or her account. Consequently, most business organizations include a credit department which must reach a decision on the credit worthiness of each prospective customer.

11. Behavior evoked by brain stimulation is sensitive to environmental changes, even in animals. Gibbons attacked their cage mates in a Yale laboratory when their brains were stimulated. The same animals, when moved to Bermuda and placed in a large corral, did not behave aggressively at all in response to the same stimulation.

Answers and brief notes:

1. Description. Note "and."
2. Description. Note "and."
3. Explanation. Note "because."
4. Argument. Note "So."
5. Description. "since" used in a temporal sense here, not functioning as an indicator expression.
6. Argument. Note "but" and "Thus."
7. Description.
8. Argument. Note "thus" and "since."
9. Description.
10. Explanation. Note "Consequently."
11. Argument. The first statement is the conclusion of this argumentative passage.