THE COMPARATIVE ANALYSIS OF APPROXIMATE NUMERICAL SOLUTIONS OF THE NAVIER-STOKES EQUATIONS

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Abstract

The range of applicability and accuracy of numerical solutions of the different approximations of the Navier-Stokes equations (local stagnation streamline approximation, "parabolized" equations, and the thin-viscous-shock-layer approach) have been analyzed to study the perfect-gas and nonequilibrium viscous flows near blunt bodies. The usage of these approximations allowed to reduce the time of calculations by factor of 5-10 in comparison with the time needed to solve the full *system* of the Navier-Stokes equations. The study demonstrates a significant influence of the regularization procedure on the approximate solutions obtained. Numerical solutions are compared with experimental data and the results of the direct simulation Monte-Carlo technique.

Nomenclature

A	$=RT_o/u_{\infty}^2$, constant, Eq. (6)
а	= radius of blunt body nose
C_f	= friction coefficient
Ε	= streamwiseflux vector
e	= specific rotational energy
h	= enthalpy
M_{∞}	= Mach number
n	= normal coordinate
Р	= pressure
Pr	= Prandtl number
q	= heat flux
R	= gas constant

 $Re_o = \rho_a \mu_a a / \mu(T_o)$, Reynolds number

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S	= coordinate along the body generatrix
Sc	= Schmidt number
St	= Stanton number
Т	= temperature
t _w	$=T_{v}/T_{0}$, temperature factor
и	= velocity component along coordinates
v	= normal velocity component
a	= angle of attack
β	= angle of flap inclination
Y	= specific heat ratio
Θ	= $Re_s(\rho_s/\rho_s)^{\prime\prime}$, pressure correlation parameter
μ	= viscosity
Р	= density
ω	= viscosity parameter, Eq. (8)

 φ = azimuthal angle

subscripts

s	= parameter behind the shock wave
w	= wall parameter
0	= stagnation parameter
I	= first terms in Eqs. (1) - (8)
2	= second terms in Eqs. $(3) - (8)$
<i>∞</i>	= upstream parameter

Introduction

In the **past** years a lot of excellent numerical methods''' have been developed to study flowfield parameters around hypersonic vehicles in the rarefied atmosphere of the planets as well as to calculate heat fluxes and aerodynamic forces. **A** broad range of streamlining regimes versus the degree of media rarefication has been discussed by Cheng⁶, Wuest⁷, and Probstein and Kemp⁸.

This study analyzes the flowfield around a sphere and a plate with cylindrical blunting under different angles of attack. The study is based on the numerical solutions of the full system of the Navier-Stokes equations^{9,10} and different approximation approaches such as the local approximation for

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the description of the flow near the stagnation streamline^{8,11}, "parabolized" Navier-Stokes equations^{1,2,12}, and approximation of a thin viscous shock layer (TVSL)¹³⁻¹⁷. Special attentionis paid to the regime of flow for which dissipation effects of viscosity, heat conductivity, diffusion, and rotational-translational relaxation become a significant factor in the flowfield

Studies of the above mentioned problems by Cheng⁶, Probstein and Kemp⁸, Molodtsov and Riabov¹⁰, Gusev et al.¹⁸, Kogan,¹⁹ and Moss et al.²⁰ indicate that the analysis of the flow using the Navier-Stokes equations may be applicable far beyond the theoretical assumptions.¹⁹ However, in order to obtain a correct solution to the problem of streamlining the body under the flow conditions at moderate Reynolds numbers, it is necessary to accept boundary conditions of slip, temperature jump, and rotational energy jump as well as the slip of diffusion velocity at the wall.^{9,10,19,21} An inaccurate type of boundary conditions may bring significant errors which can be noticed, for example, in the results of Vogenitz and Takata,²² The authors of Ref. 22, comparing the results of numerical calculations using the DSMC method with the data for velocity obtained with the help of the simplified equations for the mechanics of the continuous medium,²³ came to the conclusion that the Navier-Stokes equations are unapplicable for the description of rarefied gas flow.

Studies of Molodtsov and Riabov,^{9,10} Molodtsov,²¹ Jain and Adimurthy,²⁴ and Moss et al.²⁰ demonstrate that consideration of slip and temperature jump conditions for the rarefied gas leads to complete agreement with the results of the study of Vogenitz and Takata,²² and also agrees with numerous experimental data measured by Russell^{ZS} under the flow conditions at Knudsen numbers Kn < 0.5. At present, many authors (see Refs. 1-5, 9-11) approach the problem of viscous perfect gas flow around a blunt body using numerical research methods based on the Navier-Stokes equations.

This study is conducted with the help of a standard finitedifference scheme which approximates the complete system of Navier-Stokes equations, and is the further modification of the scheme in the study of Molodtsov'' and Molodtsov and Riabov.^{9,10}

The study of the problem in the application of Navier-Stokes equations for the description of the rarefied gas flow around the sphere was conducted by comparing the results of the numerical calculations with the data using the DSMC technique, and also with numerous experimental data of pressure in the front critical point, the rotational temperature of the molecular nitrogen, the density distribution at the stagnation point of sphere, and heat fluxes at the cooled surface.

The Navier-Stokes Equations and Boundary Conditions

The dimensionless differential equations describing the viscous compressible flows were presented in Ref. 10. The divergent form of the Navier-Stokes equations connected with the arbitrary curvilinear coordinate system has been used. To describe the molecular gas flow with rotational-translational relaxation, the system of the equations in Refs. 9, 10 should be added by a relaxation equation, a state equation, and by expressions for total energy, heat flux, and the rotational energy diffusion flux.²⁶

On the outer boundary of the computational region the gas flow was assumed to be undisturbed. We assumed fulfillment of the conditions of "free flow" (see Refs. 3, 9, 21, and 26) at the **distances** far from the body. **On** the central streamline, the symmetry condition of the flow was used. On the surface of the body we specified the conditions of the slip, the temperaturejump **as** well as the rotational temperature jump, and the diffusion velocity slip. These expressions and accommodation coefficients are given in Refs. 10, 11, 19, 21, and 26.

Numerical Method

The numerical investigation was made by means of the conservative finite-difference scheme developed by Molodtsov'' and Molodtsov and Riabov^{9:10} The difference approximation of viscous stress tensor components, heat flux vector, and velocity components normal to the curvilinear surface has been made by symmetrical formulae. The convective terms of the equations are approximated by nonsymmetrical formulae of the second order which seem to have been examined for the diffusion-convective equation in Refs. 9 and **28**.

A stationary solution of the problem is executed by iteration schemes like the alternating direction implicit technique^{1,2} and Seidel's method.^{3,9,10,21}

Numerical Results

Perfect Gas

A comparison of the numerical results performed with the usage of the Navier-Stokes equations^{9,10,21,26,27} and of the DSMC technique^{22,29} was made by Molodtsov and Riabov.^{9,10} The data presented there agree satisfactorily with the DSMC calculations and testify to the validity of the calculated results for a weakly rarefied gas flow near a sphere obtained using the Navier-Stokes equations at small Reynolds numbers **R**e,.

As an example, the results of numerical solution of the system of the Navier-Stokes equations are shown in Figs. 1, 2 for streamlining supersonic viscous flow of perfect diatomic gas (solid lines) near a sphere at upstream parameters: Re, = 14.4, $M_w = 6.5$, $\omega = 0.75$, Pr = 0.72, $\gamma = 1.4$, and $t_w = 0.3$. The boundary conditions on the body surface were the conditions of adhesion.

The distribution of the flow parameters near the sphere is presented in Fig. 3 in the case of streamlining thermal isolated sphere by monatomic viscous perfect gas flow at the following parameters: Re, = 26, M_{w} = 10, y = 5/3, ω = 0.5, Pr = 0.75, q_{w} = 0.

Rotational-Translational Nonequilibrium Viscous Flow

The investigation of the polyatomic rarefied gas flow around bodies requires accounting for the nonequilibrium energy transfer between translational and rotational degrees of freedom.^{6,19,26} Departure from rotational-translational equilibrium on the flow structure near a sphere of radius a = 0.3 m can be expected for flight at altitudes 90-120km (see Ref 7). In the experimental simulation of these flow regimes in wind tunnels, the nonequilibrium nature of the energy transfer must always be taken into **account**.³³⁻³⁶ The nonequilibrium excitation of the rotational degrees of freedom of the molecules can lead to **an** increase in the translational temperature and the thickness of the disturbed zone compared with the equilibrium case for y = 1.4 (seeRef. 26).

As an example, displayed in Fig.4a are the distributions of the translational (T_1) and equilibrium (T_{eq}) temperatures, and the nonequilibrium rotational energy (e,) at Re, = 14.4, M_w = 6.5, $\omega = 0.75$, Pr = 0.67, Sc = 0.75, $\gamma = 5/3$, $t_w = 0.3$. The distribution of pressure (p_1) and velocity (v_r) in the vicinity of the stagnation streamline is shown in Fig. 4b (solid lines). The rotational degrees of freedom in the upstream flow were assumed to be in equilibrium. The nonequilibrium character of the rotational-translational energy exchange mainly appeared in the considerable difference between the distribution of rotational and translational temperatures, which is proved by experiment of Ahouse and Bagdonoff³⁴ and Tirumalesa³⁶. The parameters of pressure and velocity (see Fig. 4b) as well as density (Fig. 2) are not sensitive very much to the type of energy exchange between translational and rotational degrees of freedom. Also, it was mentioned by **Riabov**,²⁶ that the influence of the rotational-translational nonequilibrium in the upstream flow is only concentrated in the front region of the viscous shock layer.

For the considered flow regimes, the parametes of the Stanton number *St* (squares), as a function of Reynolds number *Re*, for the stagnation point at ground-testing values of the temperature factor $t_w = 0.3$ *l5* are presented in Fig. 5. The comparison of the numerical results, considering (black squares) and not considering (light squares) slip and temperature jump indicates significant errors of non-slip heat-flux data at *Re*, < 15. In previous studies^{14,26} it has been found that these numerical results are in good agreement with experimental data" received in the vacuum wind tunnel by the method of thermal sensitive coatings.³⁰

It was noticed by many, researchers (see bibliography in Ref. 31) using pressure probes for measuring the parameters of supersonic rarefied gas flow, that a significant increase in the measured pressure p_w over the value p_o 'becomes evident when the value of the Reynolds number Re, gets smaller. In order to exclude the influence of Mach number M_w when its value is small, and also to exclude parameter y, parameter $\Theta = Re_i(\rho/\rho_w)^w$ (see Ref. 31) was used with the purpose of correlating pressure data at the stagnation point in the transitional regime. All the experimental data³¹ for the considered shape of the probe nose correlate within the accuracy of \mathcal{R} . Another interesting fact is that the ratio of the pressures p_w/p_o 'can be less then 1 in a certain range of the values of parameters \mathcal{R} , and then as \mathcal{R} decreases, the ratio begins to increase.

It was found by Chue³¹, that this phenomenon occurs both on cooled as well as on thermally isolated probe surfaces, and in the last case the quantity of the difference is higher. It was also noted that a monatomic gas has a tendency for a lower level than a diatomic gas. In Fig. 6 the numerical results for gases with different specific heat ratio y and different probe surfacetemperatures are presented. The calculation was done using parameters changing in the range of 6 < Re, < 180; $3.8 < M_{\infty} < 12$; $0.19 \text{ }_{\text{w}} < t < 1$. The comparison⁹ of the experimental³¹ and computational data favorably indicates a satisfactory correlation between them.

$$u = u_1(n) \sin(s)$$
 (2)

Local Approximate Solutions of the Navier-Stokes equations

The solution of the complete system of Navier-Stokes equations, relaxation equations, and chemical kinetic equations, which describe nonequilibrium viscous flows near hypersonic vehicles, demands spending a lot of time for computations.^{1,2} Simplification of the initial system of equations drastically decreases the amount of time.

At present, some approaches, which suggest a local similar character of flow near the stagnation streamline, are known. They are based on modification of the Navier-Stokes equations to the system of nonlinear ordinary differential equations (see Refs. 8, 11, 23, 24, and 37-39). This study, using the assumptions of Probstein and Kemp,⁸ discusses one of many possible variants of structuring a local similar solution. A proof of applicability of such approach is presented by comparing the solutions with the numerical calculations of the complete system of the Navier-Stokes equations. The calculations of this study, done for the perfect gas and viscous relaxing diatomic gas flow, taking into consideration rotational-translational nonequilibrium. The same technique can be developed to receive local similar solutions for the study of hypersonic flows with physical and chemical processes.

The system of the full Navier-Stokes equations in an orthogonal curvilinear coordinate system \mathbf{s} , n, φ (s is the coordinate measured along the generatrix of the body in the meridional plane, n is the normal to the surface of the body, and φ is the azimuthal angle) is derived in Refs. 3, 9, 10, and 21 in divergence form for axisymmetric flow of a perfect **gas**. To describe the flow of diatomic gas with allowance for rotational-translational relaxation, the system must be augmented by a relaxation equation, the equation of state, expressions for total energy of *unit* mass of the gas, the heat flux, and the diffusion flux of the rotational energy (see Ref. **26**). The following system of modification of the similarity expressions should be introduced:

$$\rho = \rho_1(n); v = v_1(n) \cos(s)$$
 (1)

$$T = T_{1}(n) + \frac{T_{2}(n)}{2} \sin^{2}(s)$$
 (3)

$$e_i = e_1 (n) + \frac{e_2(n)}{2} \sin^2(s)$$
 (4)

$$p=p_1(n) + \frac{p_2(n)}{2}\sin^2(s)$$
 (5)

$$p_1 = \rho_1 A T_1; \quad p_2 = \rho_1 A T_2$$
 (6)

$$\mu = \mu_1(n) + \frac{\mu_2(n)}{2} \sin^2(s)$$
 (7)

$$\mu_1 = T_1^{\omega}; \quad \mu_2 = \omega \mu_1 \frac{T_2}{T_1}$$
 (8)

The analysis of Euler equations, describing nonviscous gas flow, indicates, that having selected the functional dependency for velocity components from the Bernoulli integral, temperature should be presented as in expressions (1)-(8), as it was made in Refs. 8, 11, and 38. The latter was not considered in Refs. 24, 37-39 that brought to unjustified simplification of equations for the function $p_2(n)$, which did not contain "viscous" terms in an obvious form.

Substituting the Eqs. (1)-(8) in the initial system of the full Navier-Stokes equations¹⁰ written along line s = 0, we will get the system of equations (see Ref. 11) for the functions ρ_1 , v_1 , T, e,. In order to receive equations for the functions u_1 , T_2 , and e, let us consider expressions of the Navier-Stokes equations as functions of the variable s and differentiate them required number of times for even or odd functions. Then substituting the Eqs. (1)-(8) in the arrived equations, we will get the subsystem of equations for the functions u_1 , T_2 , e_2 (see Ref. 11). To solve the obtained system of differential equations, a onedimensional variant of the implicit scheme^{9,10,21} was used. For the initial distributions of the flow functions, there were selected values of the functions corresponding to undisturbed upstream flow or profiles, received earlier with approximately the same main flow parameters. This computational method is free from the necessity of disclosure of peculiarities which occur in the flow field.

Figs. 1 and 2 presents the results of numerical solution of the system of local approximate equations for streathlining supersonic viscous flow of perfect gas (dashed lines) near sphere at upstream parameters: $Re_{,} = 14.4$, $M_{\omega} = 6.5$, $\omega = 0.75$, Pr = 0.72, $\gamma = 1.4$, $t_{\omega} = 0.3$. The boundary conditions on the body surface were the conditions of adhesion. There are also the results of the full Navier-Stocks equations (solid lines). Comparison of the results points to the acceptable accuracy of the results, obtained while assuming local similarity character of the flow near the axis of symmetry of smooth blunt body. Some differences (up to 10-15%) are noticed in the shock wave in front of the body. In order to describe the flow in this zone, probably it is necessary to consider approximation of higher level. The significant differences are in the distribution of u_1 (see Fig. 1a).

Fig. 4 presents the results of the calculations using approximation method (dashed lines), and the solutions of the full Navier-Stokes equations and the relaxation equation (solid lines) of streamlining the sphere by rotational exciting nitrogen. It is **assumed** that in the upstream flow, the rotational degrees of freedom are in equilibrium with the translational ones, and $Re_{,} = 14.4, M_{w} = 6.5, \omega = 0.75, Pr = 0.67, Sc = 0.75, \gamma = 5/3, t_{w} = 0.3$. Presented comparison demonstrates that the local approximation technique, based on transformation (1)-(8), is applicable for the description of the viscous relaxation gas flow. The consideration of the nonequilibrium character of rotational-translational energy exchange under this regime of streamlining leads to significant differences of T_{I} and $e_{,}$ from their equilibrium values T_{eq} (see, also, Refs. 10 and 26).

The results of this study were compared with the data of Levinskey and Yoshihara²³ for the case of streamlining thermal isolated sphere by viscous perfect gas flow at the following parameters: $Re_o = 26$, $M_w = 10$, $\gamma = 5/3$, $\omega = 0.5$, Pr = 0.75, $q_w = 0$ (see Fig. 3). Symbols are the same as in Fig. 1. Though there is a good correlation of the results with the data in Ref. 23 (dark and light points), the number of differences

is significant. One of the reasons for such differences may be an assumption about the thinness of the compressed viscous shock layer at hypersonic velocity of the upstream flow which was considered in Ref. 23.

Applications of "Parabolized'Navier-Stokes Equations

One of the ways of economic usage of the computer resources is to create a new marching method of solving simplified ("parabolized") Navier-Stokes equations (see Refs. 1, 2, 12, and 40). These equations are usually obtained from a complete system of the equations by means of exclusion of derivatives from viscous terms in the marching direction (see Refs. 4 1-43), while preserving the elliptical type of equations in crossflow directions. This procedure permits qualitative study of flows with crossflow separation if streamwise separation is omitted (see Refs. 44 and 45).

In this study, the research of viscous perfect gas flow near blunt bodies was done with the means of iterational procedure of the Seidel's technique,^{9,10} simplified for "parabolized" Navier-Stokes equations. The *study* is based on the concept of the regularity (in terms of Ref. 46) of the formulation of initialboundary value problem.

As a test, a problem was solved for streamlining of the sphere by supersonic perfect gas flow at Reynolds number Re, = 14.4. The variations of the selection of the approach are illustrated by the solving of the problem of streamlining of the finite thickness plate, at different angles of attack.

In this study the momentum equation was written in the projection of the axis of Cartesian coordinate system. While obtaining simplified equations, it was assumed that the streamwise components of viscous stress tensor were small compared to normal and azimuthal components.

In Refs. 43-45 it was noted that Cauchi problem is irregular for the system of Navier-Stokes equations, simplified in this way, with the initial data at fixed value of the streamwise coordinate x = const, in subsonic flow region, where Mach number M_x was calculated using streamwise velocity component, and $M_x < 1$. Following the Vigneron's technique,⁴⁶ we introduce the vector \mathbf{E}^* as the resultant streamwise flux, and the vector \mathbf{E}^p as the portion of the original streamwise flux responsible for introducing ellipticity into the equations through the subsonic boundary layer. The superscript asterisk denotes the omission of streamwise viscous derivatives. We consider two cases of approximation for $\partial \mathbf{E}^p/\partial s$: **A**) it is equal zero, and B) it was downstream extrapolated. The details are

in Ref. 12.

The finite-difference scheme (Refs. 9, 10, and 21) was modified considering the distribution of disturbances downstream. This led to the decrease of the order of approximation of convective components in streamwise direction up to the first order. The solutions of the full Navier-Stokes equations were used both for the test calculations as well as for the obtaining of the initial conditions for the Cauchi problem.

The problem of streamlining of the sphere by supersonic viscous perfect gas flow was solved in order to test the selected method of the regularization of irregular Cauchi problem for "parabolized" Navier-Stocks equations. The following are the upstream flow parameters: $He_r = 14.4$, $M_w = 6.5$, $t_w = 0.34$, $\gamma = 1.4$, $\omega = 0.85$. Differential gnd 24 x 24 was used in the calculations.

The distributions of pressure $p/\rho_{\omega}\mu_{\omega}^2$, heat flux $q/\rho_{\omega}\mu_{\omega}^3$, and friction coefficient c_f along the sphere generatrix *sla* are presented in Figs 7 and 8. The profiles of pressure, density ρ/ρ_{ω} , enthalpy $h = c_p T/u_{\omega}^2$, and streamwise velocity component u/u_{ω} along the normal n/a at s = 58.5°, based on the calculation of the full Navier-Stokes equations (solid lines) are also shown in Figs. 7 and 8. The results of the calculation using the simplified system of equations are presented by dashed lines (case A), and by dot-dashed lines (case B of downstream extrapolation). The same symbols and lines are used in the further figures.

As the comparison demonstrates, the data obtained from different models favorably agree with each other. The usage of gradient pressure downstream extrapolation (case B) offers the results which are the closest to the results of the calculations using the full Navier-Stokes equations in this case. It is noted that the time of the calculations of "parabolized" equations approximately 5 times smaller than of the full system of the Navier-Stokes equations.

The computational method for the system of "parabolized" Navier-Stokes equations was used in the study of the flow near the plate with cylindrical blunt at the angle of attack towards upstream flow. The zone of blunting was calculated using the full Navier-Stokes equations, and the flow field below the conjugate point was calculated using the simplified method. It was assumed that the stagnation point of the flow coincides with the stagnation pomt on the surface of the cylinder. Upstream flow parameters and the characteristics of the computational gnd are the same as mentioned above.

The results of the calculation of the streamlining of the

cylindrical part of the body are presented in the Figs. 9-11 by solid lines, and the results for the flat forward surface by dashed lines (case **A**) and dot-dashed lines (case **B**) starting from the conjugate point marked by cross there. The results for leeward side at $a = 18^{\circ}$ are presented by dotted lines. In Figs. 9-11 are presented the distributions of pressure, friction coefficient, and heat flux along the generatrix of the plate with cylindrical blunt at different angles of attack $a = 0, 18^{\circ}, 36^{\circ}$. The distance s/a is measured along the generatrix from the stagnation point of the cylinder.

In **all** cases considered, stabilization of pressure, and density can be observed at the distances from the critical point. The selection of the regularization method significantly influences the values of pressure and friction coefficient near the conjugate point. In the case B of extrapolation, large abnormal values of pressure (see Fig. 9) are noticed, and also diminished data for the fiction coefficient values are observed in the studied region of flowfield (see Fig. 10).

Nonmonotonous character in pressure and density distributions at large angles of attack $a > 30^{\circ}$ is discovered (see Fig. 9). The significant increase of subsonic flowfield zone begins **as** further increase of the angle of attack occurs. This causes the significant emergence of the ellipticity properties in streamwise flow direction. The system of full Navier-Stokes equations should be used under these conditions. The latter did not allow to receive the solution in the case of $\partial E^{p}/\partial s = 0$ at $a > 36^{\circ}$. No specific features were discovered at the leeward side. The selection of the methods of the regularization has **an** insignificant influence on the value of heat flux towards the surface of the body (see Fig. 11).

The above results were used for study of the gas flow near a long plate with flap. The profiles of the flow parameters as the initial conditions along normal towards the plate were obtained using "parabolized" Navier-Stokes equations at s(x)= const. In this case $Re_r = 14.4$, $a = 18^\circ$, angle of the flap declination is $\beta = 15^\circ$. The considered flowfield area was limited by the body surface, upstream boundary at s/a = 8.95, and downstream boundary at s/a = 12.26. Cartesian coordinate system adjacent to the plate surface wes used. The upper boundary and the profile of the body were calculated using the formulas in Ref. 47. The location of the upper boundary was selected in a way that the disturbances from the body would not reach the boundary. The conditions of zero gradients of the soughtfunctions were accepted at the downstream boundary.

The computational results (see Fig. 12) of flow parameters

near the flap allowed the establishment of monotonous increase of pressure and density along the plate with the flap. But the distribution of the heat flux and friction coefficient has strong nonmonotonous character. This qualitative effect has also been mentioned by Davis and Rubin.⁴⁰

The considered results indicate that using simplified ("parabolized") Navier-Stokes equations for the calculations of flowfield parameters near the blunt bodies is necessary to select the regularization procedure for obtaining solutions in subsonic areas. This is important for the description of the flow near the conjugate points and at large angles of attack. In studies of streamlining smooth bodies at small angle of attack and the sphere, the regularization procedure of pressure gradient extrapolation in marching direction can be used.

Approximation of a Thin Viscous Shock Laver

The model of a thin viscous shock layer (TVSL) is broadly used for the description of the structure of nonequilibrium viscous gas flow near blunt bodies or hypersonic vehicles (see Refs. 6, 13-15). In the present study, a comparison between the results from the TVSL (dashed lines) model and the solutions of the complete system of the Navier-Stokes equations with no slip (solid lines) boundary conditions is made and shown in Fig. 13. The parameters of upstream perfect gas flow were the following: $Re_{1} = 7.33$, $M_{2} = 6.5$, t_{w} = 0.315, γ = 1.4. The largest difference in the distributions of parameter; calculated from the TVSL model at low Reynolds numbers Re, was observed far away from the wall of the sphere where the condition, p = const, no longer holds along the central streamline. The agreement between the two sets of data'on the distribution of parameters over the surface of the body itself is rather satisfactory (see also Ref. 14). All this indicates that it is possible to use the model of a thin viscous shock layer in investigating the hypersonic flow over blunt bodies in the transition region at low Reynolds numbers Re..

The calculated Stanton numbers *St* as a function of *Re*,, for the stagnation point at in-flight values of the temperature factor $t_w = 0.033$ are presented in Fig. 5 (solid line: the TVSL solutions for perfect gas; dashed line: results for second order boundary layer approximation,¹⁴ the Navier-Stokes equation solutions with nonslip (light squares) and slip (dark squares) boundary conditions). The results correlate well with the expenmental data.²⁷ The analyses of nonequilibrium flows in a TVSL at different flight conditions were made in previous papers (see Refs. 14, 15, and 47).

Concluding Remarks

The results of this study **confii** the hypothesis about the applicability of the Navier-Stokes equations as well as local approximation equations, "parabolized" Navier-Stokes equations, and approximation of a thin viscous shock layer for the description of rarefied gas flow near the simple shaped bodies. This conclusion is correct for perfect gas and for rotational-translational nonequilibrium and chemical nonequilibrium gas flow.

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Fig. 1 a) Velocity components u_1 , v_1 , and b) pressure functions p_1 , p_2 at critical stagnation Line of the sphere: solid lines - solutions of the Navier-Stokes Eqs., dashed Lines - local approximation.



Fig. 2 Density ρ_1 and temperature T_1 at critical stagnation line of the sphere: solid lines - solutions of the Navier-Stokes Eqs., dashed lines - local approximation, dot-dashed lines - the case of translational-rotational nonequilibrium.



Fig. 3 Comparison of present results with data of Ref. 23: a) u_1 , v_1 , and ρ_1 ; b) T_1 , p_1 , and p_2 .



Fig. **4** a) Translational T, and equilibrium T_{eq} temperatures, rotational energy **e**, and b) velocity v_l and pressurep, at critical stagnation line of the sphere: solid linea - solutions of the Navier-Stokes Eqs., dashed lines - local approximation.



Fig. 5 Stanton numbers *St* as a function of Reynolds numbers Re; Solid line - the TVSL method¹⁴, dashed line - the second order boundary layer approximation;¹⁴ and the Navier-Stokes Eqs. solutions (squares).





Fig. 6 Pressure at the front stagnation point of the sphere.

Fig. 8 Pressurep, density ρ , enthalpy h, and streamwise velocity component *u* across the normal *n* at *s* = **58.5**" are the solutions of full (solid lines) and "parabolized" Navier-Stokes equations.



Fig. 7 Pressure p, density ρ , heat flux q, and friction coefficient c, along the sphere generatrix s are the solutions of **full** (solid lines) and "parabolized" Navier-Stokes equations.



Fig. 9 Pressure at the plate surface.





Fig. 10 Friction coefficient c_{f} along the plate surface.

Fig. **12** Heat flux q and friction coefficient c_f along the plate surface with the flap ($\beta = 15^\circ$) at angle of attack **a** = **18°**.



Fig **11** Neat flux q at the plate surface.



Fig. 13 Pressurep, temperature T, and density ρ at the stagnation streamline of the sphere: the TVSL model' (dashed lines), and the solutions of the Navier-Stokes Eqs. (solid lines).