

# Numerical analysis of compression and expansion binary gas-mixture flows

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**AIAA, Aerospace Sciences Meeting and Exhibit, 34th, Reno, NV, Jan. 15-18, 1996**

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**NUMERICAL ANALYSIS OF COMPRESSION AND EXPANSION  
BINARY GAS-MIXTURE FLOWS**

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Abstract

Diffusion effects in the flows of compression and expansion have been studied using the DSMC technique and numerical solutions of the Navier-Stokes equations. The range of applicability of these techniques has been identified using the comparison of the results with experimental data. Similarity analysis was used to study the flow structure of Ar-He, N<sub>2</sub>-H<sub>2</sub>, and O<sub>2</sub>-N<sub>2</sub> mixtures. Diffusive effects are becoming significant in the area of the shock wave, near the stagnation point, and in the area of high pressure. The effects have major influence on the shock-wave width, adiabatic temperature, pressure at the stagnation point, and on the effectiveness of species separation and ambient gas penetration in compression and expansion flows. Comparison of the DSMC results, the Navier-Stokes solutions, and experimental data indicates the areas of applicability of the continuum concept for studying diffusive effects in low-density gas-mixtures.

Nomenclature

$a$	= radius of a sphere
$C_f$	= $\tau_w / \frac{1}{2} \rho_\infty U_\infty^2$ , friction coefficient
$C_p$	= $(p - p_{invisc}) / \frac{1}{2} \rho_\infty U_\infty^2$ , Homann's pressure coefficient
$C_p'$	= $(p - p_\infty) / \frac{1}{2} \rho_\infty U_\infty^2$ , pressure coefficient
$Kn$	= Knudsen number
$K_2$	= similarity parameter
$l$	= length scale parameter at infinity
$M$	= Mach number

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$m$	= mass of a molecule
$n$	= coordinate along the normal of a sphere
$p$	= pressure
$r$	= radius or spherical coordinate
$Re_0$	= $\rho_\infty U_\infty a / \mu(T_0)$ , Reynolds number
$s$	= coordinate along the surface of a sphere
$T$	= temperature
$U$	= stream velocity
$x, y$	= Cartesian coordinates
$\alpha_f$	= separation parameter
$\mu$	= viscosity coefficient
$\rho$	= density
$\tau$	= viscous stress tensor
$\omega$	= temperature exponent of the coefficient of viscosity

subscripts

$w$	= wall conditions
1	= heavy species
2	= light species
$\infty$	= upstream parameter
*	= parameter at critical conditions ( $M = 1$ )
+	= extreme value of a parameter

Introduction

Diffusive processes have a significant effect on the structure of a low-density gas mixture flow near blunt bodies<sup>1,2</sup> and in an underexpanded free jets.<sup>3-6</sup> The effect of abnormal increasing of the temperature recovery factor at the stagnation point of a blunt body in the rarefied gas mixture flow was studied experimentally by Maise and Fenn,<sup>7</sup> and Bochkarev et al.<sup>8,9</sup> The structure of rarefied gas mixture flows about a sphere was analyzed by Riabov<sup>10</sup> and Molodtsov and Riabov<sup>11</sup> using numerical solutions of the Navier-Stokes equations. The normal

shock wave structure in binary gas mixture was studied by Center,<sup>12</sup> Abe and Oguchi,<sup>13</sup> Harnet and Muntz,<sup>14</sup> and Bochkarev et al.<sup>15,16</sup> The Direct Simulation Monte-Carlo (DSMC) technique was used by Bird<sup>17-19</sup> to study the flow.

The expansion of a binary gas mixture into a flooded space was studied by Sherman,<sup>3</sup> Bochkarev et al.,<sup>4,5</sup> and others. Separation processes in the axisymmetric underexpanded jets were analyzed by Skovorodko and Chekmarev,<sup>20</sup> Gusev and Riabov,<sup>21</sup> and Riabov<sup>22</sup> in terms of numerical and asymptotic solutions of the Navier-Stokes equations. The results could be effectively used for simulating hypersonic vehicle flight in the atmospheres of planets<sup>22</sup> as well as for the separation of gas mixtures or isotopes.<sup>21</sup> Experimental data of the diffusive separation of binary mixtures were found by Rothe<sup>23</sup> and Bochkarev et al.<sup>24</sup> The phenomenon of background gas penetration into underexpanded free jets was described by Skovorodko and Chekmarev,<sup>20</sup> Gusev and Riabov,<sup>21</sup> and Brook et al.<sup>6</sup> The systematic analysis of the diffusive processes based on the DSMC technique was not provided. Also the problem of applicability of the Navier-Stokes equations to describe these processes is not solved yet.

### DSMC Method

The DSMC method has been used in this study as a major numerical simulation technique for low-density binary gas-mixture flows. The basic principles of the technique were described by Bird.<sup>18,19</sup> Both one-dimensional<sup>19</sup> and two-dimensional<sup>25</sup> DSMC codes are used in various parts of the study. In both codes, molecular collisions are modeled using the variable hard sphere (VHS) molecular model.<sup>26</sup> The Larsen-Borgnakke statistical model<sup>27</sup> is used for modeling the energy exchange between the kinetic and internal modes. The parameters of the VHS molecular model for nitrogen, hydrogen, oxygen, argon, and helium are the same as described by Bird.<sup>19</sup> The gas-surface interactions are assumed to be fully diffuse with full momentum and energy accommodation.

### Continuum Numerical Methods

Continuum regimes of the two-dimensional flows were studied by the continuum method developed by Molodtsov and Riabov,<sup>11,28</sup> which numerically solves the Navier-Stokes equations for two-dimensional (axisymmetric) flows. On the outer boundary of the computational region the gas flow was assumed to be undisturbed. We assumed fulfillment of the conditions of "free flow"<sup>11,28,29</sup> at the distances far from the body. On the central streamline, the symmetry condition of the flow was used. On the surface of the body we specified the conditions of the slip, the temperature jump as well as the rotational temperature jump, and the diffusion velocity slip. These expressions and accommodation coefficients are given in Refs. 10, 11, 28, and 29. The numerical investigation was made by means of the conservative finite-difference scheme.<sup>15,28,29</sup> The difference approximation of viscous stress tensor components, heat flux vector, and velocity components normal to the curvilinear surface is made by symmetrical formulae. The convective terms of the equations are approximated by nonsymmetrical formulae of the second order which to have been examined for the diffusion-convective equation by Price et al.,<sup>30</sup> Molodtsov,<sup>31</sup> and Molodtsov and Riabov.<sup>15,28</sup> A stationary solution of the problem is executed by iteration schemes like the alternating direction implicit technique and the Seidel's technique. All variants were calculated using the grid with 21 nodes along the normal and 31 nodes along the surface of the body.

The spherical expansion of a binary gas mixture has been studied using one-dimensional algorithm developed by Gusev and Zhabkova<sup>32</sup> and Gusev and Riabov.<sup>10</sup>

### Diffusive Effects in Gas Mixture Flow around a Sphere

Diffusive processes have a significant effect on the structure of a supersonic gas mixture flow over blunt bodies.<sup>1,2,10,11,19</sup> Compared to a monocomponent gas, the shock layer thickness increases, the mixture enrichment by heavy particles occurs in high pressure regions, and the adiabatic wall temperature rises. The noted features are illustrated by numerical results presented in this study for a flow of the nitrogen-hydrogen mixture ( $f_{N_2, \infty} = 0.1$ ) near a sphere for different regimes at Reynolds numbers  $6 \leq Re_{a,r} \leq 100$  (or Knudsen numbers  $0.257 \geq Kn_{\infty} \geq$

0.015). The Reynolds and Knudsen numbers are associated with the radius of a sphere  $a$ . The Mach number contours in the flow at  $M_\infty = 6.6$  past a sphere are shown in Figs. 1a - 1d. At  $Re_0 \leq 15$ , the regime of the fully merged layer<sup>33</sup> can be characterized by relatively smooth changing of the flow parameters and their derivatives in the flowfield. The viscous layer regime<sup>33</sup> is realized at  $Re_0 \geq 45$  (see Figs. 1c and 1d). Under these conditions the strong shock wave near the sphere causes significant gradients of the flow parameters. As a result, the major changing of species concentrations occurs in these regions. The later is illustrated in Figs. 2a - 2d for considered flow regimes.

Diffusive effects are becoming significant in the area of the shock wave, where the enrichment of the mixture with light component occurs (see Figs. 2 and 3), near the front critical point, and in the area of high pressure. The latter points to the domineering role of the baro-diffusion while components separate. The profiles of nitrogen concentration  $f_{N_2}$  are very sensitive to the pressure gradients at the stagnation stream-line near a sphere, which are different for the fully merged layer and viscous layer regimes shown in Figs. 3a and 3b.

The dissipative viscous and diffusion processes have the major influence on the distribution of the temperature: the overall temperature  $T$  and translational temperature  $T_t$  can be significantly higher than the stagnation temperature  $T_0$  in the flow (see Fig. 4). It indicates that at the stagnation point the gas temperature slightly increases near the thermoisolated wall. This effect was considered in detail by Maise and Fenn,<sup>7</sup> Bochkarev et al.,<sup>9</sup> and Molodtsov and Riabov.<sup>11</sup>

In rarefied diatomic gas flows, non-equilibrium processes should be accurately considered. For fully-merged-layer flow regime at  $Re_0 = 15$ , the rotational-translational relaxation occurs in the total area of the flow near the stagnation point. In viscous-layer regime at  $Re_0 = 100$ , nonequilibrium effects are located in the shock wave and in the area behind it (see Fig. 4).

The profiles of the stream velocity  $U$  and diffusion velocities in the flow of nitrogen-hydrogen mixture at the stagnation stream-line near a sphere are shown in Figs. 5a - 5d for different flow regimes. As the rarefaction of gas media decreases (or Reynolds number  $Re_0$  increases) the structure of the diffusive zones changes significantly. At

high values of the Reynolds number, the maximum magnitude of the diffusion velocity of heavy component (nitrogen) correlates with the minimum magnitude of the diffusion velocity of light component (hydrogen), and it corresponds to the maximum concentration of light species in the shock wave near a sphere (see, also, Fig. 3a).

Translational relaxation and diffusion are the major factors of difference between the heavy ( $T_1$ ) and light ( $T_2$ ) component overall temperatures in the flow of nitrogen-hydrogen mixture near a sphere. The profiles of  $T_1$ ,  $T_2$ , and  $T$  are sensitive to the regime of the flow characterized by the parameter  $Re_0$ . As it is shown in Figs. 6a - 6d, the overall temperature of mixture is approximately of the same value as for the light species. The overall temperature of heavy component is significantly larger than the stagnation flow temperature  $T_0$  in the flow behind the shock wave, and its maximum magnitude is approximately greater the parameter  $T_0$  by factor of 2.5. Therefore, heavy species is a hot-gas fraction in the flow region.

The hemispherical probes have been used by many researchers<sup>34-37</sup> to measure impact pressure in the rarefied gas flows of nitrogen. The experimental data of Chang and Fenn<sup>36</sup> and Potter and Bailey<sup>35</sup> are compared with the DSMC results shown in Fig. 7. The ordinate is the pressure coefficient of Homann<sup>34</sup>  $C_p$ . The abscissa is the Reynolds number  $Re_{0,d}$  based on probe diameter. The pressure was calculated by translational temperature which was significantly larger the overall mixture temperature in the stagnation region of the flow (see, also, Fig. 4). Therefore, the calculated impact pressure parameters are higher than the measured data.

The distribution of the pressure coefficient  $C_p'$  along the spherical surface is shown in Fig. 8 for different flow regimes. These functions are approximately the same as for monocomponent-gas flow and satisfy the modified Newton's approximation law.<sup>38</sup> The corresponding values of the friction coefficient  $C_f$  on the spherical surface are presented in Fig. 9.

Only few experiments have been known in analyzing the structure of rarefied binary-mixture flows.<sup>1</sup> In Figs. 10a and 10b the DSMC results are compared with the experimental data of Bochkarev and Prikhodko<sup>16</sup> and Rebrov<sup>1</sup> concerning the nitrogen concentration profiles in the flow of nitrogen-hydrogen mixture at  $Kr_\infty = 0.025$ ,

$f_{N_2, \infty} = 0.24$  and  $M_{\infty} = 3.5$ . The numerical results are in a good agreement with the experimental data at the stagnation stream line  $s = 0$  (case a) as well as along the normal far from the line at  $s = 1.02a$  (case b). The concentration  $f_{N_2}$  of the heavy component near the sphere's critical point (see Fig. 10a) increases approximately two times its concentration in the upstream flow, and is in good correlation with the experimental values.<sup>1,2,16</sup> The reason for this increase is the baro-diffusion effect induced by high pressure gradients (see Fig. 11b). The same effect occurs in the zone of high pressure at some distance from the body. This zone (see Figs. 2 and 10b) is like a torus cloud of heavy component near the sphere. A considerable increase of concentration  $f_{N_2}$  near the body leads to an essential increase of the adiabatic wall temperature  $T_{ad}$  over the corresponding value for a monocomponent gas.<sup>11</sup>

In Figs. 10a and 10b the results of numerical solutions of the Navier-Stokes equations are presented. In both cases the DSMC results correlate with the experimental data much better than the continuum-concept solutions. The same difference of the results has been found in the analysis of temperature and pressure profiles at the stagnation stream line (see Figs. 11a - 11b). Also it was found, that the shock wave becomes broader compared to the streamlining of the surface of the sphere by the monocomponent gas (hydrogen), and at this point the gas temperature slightly increases near the wall (see Fig. 11a). The pressure parameters are significantly different because of the relaxation processes mentioned above. The Navier-Stokes equations were solved under the conditions of the one-temperature approximation and rotational-translational equilibrium.

The effect of baro-diffusion also determines the structure of the flow of the gas mixture with a small molecular mass ratio of species. The latter condition may be effectively applied, for instance, in the separation of isotopes with the help of blunt probe.<sup>39,40</sup> The numerical simulation of an argon isotope mixture flow over a thermal isolated sphere ( $Ar^{40} - Ar^{36}$ ) was studied by Molodtsov and Riabov<sup>11</sup> analyzing the numerical solutions of the Navier-Stokes equations. It was found that the separation parameter  $\alpha_f = f_1(1 - f_{1,\infty})/f_{1,\infty}(1 - f_1)$  (index 1 is refer to a heavier component) reached its maximum value  $\alpha_f = 1.033$  in the stagnation point which is near

experimental data<sup>40</sup> ( $\alpha_f = 1.045$ ). The DSMC results confirm this conclusion.

As in many aerodynamic studies the gas used in testing is air, which can be considered as the mixture of oxygen and nitrogen or "isotopes". The computation was made at  $Kn_{\infty} = 0.036$ ,  $M_{\infty} = 3.8$ . The density and heavy species (oxygen) concentration contours in air past a sphere are shown in Figs. 12a and 12b, correspondingly. At this regime the influence of the diffusive processes in the air was estimated to be about 3-5%, and it emerges in increasing the concentration of oxygen in the critical point of the spherical probe (see Fig. 13). The solutions of the Navier-Stokes equations indicate that  $\alpha_{f,max} = 1.047$ , and this is in a good agreement with the DSMC results.

#### Spherical Expansion of a Binary Gas Mixture

The spherical expansion of a binary gas mixture into a flooded space might be considered as an important theoretical model for study of separation processes in the axisymmetric jets, in the physics of explosion, in an isotope-separating technology, etc.

The problem was studied by Skovorodko and Chekmarev,<sup>20</sup> Gusev and Riabov,<sup>21</sup> and Riabov<sup>22</sup> by means of the numerical and analytical solutions of the Navier-Stokes equations. Bochkarev et al.<sup>41</sup> have investigated experimentally spherical and cylindrical rarefied gas expansion with a stationary shock wave.

Under the continuum conditions, Gusev and Riabov,<sup>21</sup> and Riabov<sup>22</sup> have found that the numerical and analytical solutions obtained for density, velocity, pressure, and temperature are similar to the solutions for a one-component gas. The major difference is in the structure of the spherical shock wave, which should be studied accurately by means of the kinetic techniques.

The fulfillment of the conditions formulated by Gusev and Mikhailov<sup>42</sup> and Gusev et al.<sup>43</sup> for the similarity of flows in strongly underexpanded viscous jets and spherically expanding flows was studied by Gusev et al.<sup>44</sup> The limits of applicability of the similarity conditions have been established on the basis of numerical and asymptotic solutions of the Navier-Stokes equations and experimental data analysis.

It was found that the spherical flow could be separated

by the coordinate  $r_+$ , at which the stream parameters are extremal, into two regions with significantly different properties. In the first "internal" region at  $r < r_+$  the flow is supersonic. In the second "external" region at  $r > r_+$  there is a transition of supersonic flow into subsonic stream at the infinity. To characterize flow parameters in the first region, we introduce two major similarity parameters of Knudsen number  $Kn$ , and  $Kn_+$ , based on the critical radius of a spherical source  $r_c$  and on the coordinate  $r_+$  estimated by the method of Gusev et al.<sup>44</sup> and Gusev and Riabov.<sup>21</sup> The parameter of Knudsen number  $Kn_\infty$  based on the length scale parameter at infinity  $l = (Qa_\infty/4\pi r p_\infty)^{1/2}$  is the major similarity parameter in the second region. Using the definition of the overall mean free path<sup>19</sup> for the mixture of the VHS molecules, the following correlations between these similarity parameters could be found:

$$\frac{Kn_\infty}{Kn_+} = \left(\frac{m_+}{m_\infty}\right)^{1/4} \left(\frac{T_\infty}{T_+}\right)^{\omega+1/4} \sqrt{\frac{p_+}{p_\infty}} \quad (1)$$

$$\frac{Kn_+}{Kn_*} = \frac{m_+}{m_*} \left(\frac{\gamma+1}{\gamma-1}\right)^{1/2} \Theta_+ \frac{2(\gamma-1)(\omega-0.5)}{[2(\gamma-1)(1-\omega)+1]} \frac{1}{Z_+} \times \left(\frac{3}{4} Re_*\right)^{\frac{[1-2(\gamma-1)(\omega-0.5)]}{[2(\gamma-1)(1-\omega)+1]}} \quad (2)$$

The parameter of Reynolds number  $Re$ , and coefficients  $\Theta_+$  and  $Z_+$  depend on parameters of the mixture.<sup>21,22</sup>

Under certain conditions, the flow regimes in the external region at  $r \sim r_+$  and  $r > r_+$  could be far from the continuum regime. Following Gusev and Mikhailov<sup>42</sup>, we introduce the similarity parameter  $K_2 = Re_*(p_\infty/p_0)^{1/2}$  to study the flow structure in this region. It is not difficult to estimate that  $Re_{*1} \approx K_2$ , and calculate the ratio of characteristic Knudsen numbers from Eq. (1).

The present study was undertaken for the purpose of clarifying the role of diffusive processes in the spherical expansion flows. The results of the DSMC calculation are presented for argon-helium mixtures at different initial component concentrations.

The argon concentration, density, pressure, temperature, and velocity ratios in a  $K_2 = 12.4$  expansion of argon-helium mixture are shown in Figs. 14a - 14c. This regime of the flow corresponds to the near-continuum one at the infinity. In supersonic region, the concentration of argon changes insignificantly. Because of the large gradients of the flow parameters in the shock wave front, the considerable increase in the velocity of the light helium component in this region is realized. In the case  $f_+ = f_\infty = 0.5$ , maximum velocity of helium component exceeds the limiting overall velocity of the mixture by more than the factor of three.

Concentration of the light component occurs in the leading front of the spherical shock wave, just as in the normal shock-wave case considered by Bird<sup>17</sup> and Zeldovich and Raizer.<sup>45</sup> The same effect was described by Nagornykh.<sup>46</sup> The enrichment of the mixture with the light component in the leading front of the wave begins with the increasing of the pressure. It indicates that baro-diffusion effect dominates in this part of the shock wave. The minimum value of argon concentration in this area correlates with the minimum magnitude of density (see Fig. 14a).

Under these conditions it is possible to assume that the flow can be considered as continuum with mono-temperature (see Fig. 14b).

At decreasing the similarity parameter  $K_2$ , the flow regime in the shock wave and behind it will change dramatically. In Figs. 15a - 15c the argon concentration, density, pressure, temperature, and velocity ratios in a  $K_2 = 1.24$  expansion of argon-helium mixture are shown. The diffusion zone is wider than in the previous case. In the region behind the shock wave, multi-temperature regime of the flow is identified. Nevertheless, the significant increasing enrichment of the mixture with the heavy component inside the wave front, described by Gusev and Riabov<sup>21</sup> by means of the continuum concept, has not been found.

Using the DSMC technique, the spherical expansion of a binary gas mixture into a flooded space has been analyzed in the case of the presence of a diffusive flux at the infinity  $r \gg r_+$ . The numerical results were calculated for the case of the expansion of helium with a slight argon content ( $f_+ = 0.011$ ) into a space filled by argon with small admixture of helium ( $f_\infty = 0.9$ ). The

distributions of the argon concentration  $f_{Ar}$ , density, pressure, temperature, and velocity ratios in such a flow with  $Re_2 = 453$  and  $K_2 = 4.53$  are presented in Figs. 16a - 16c.

The other case of the expansion of argon with slight helium content ( $f_2 = 0.99$ ) into a space filled by helium with a small admixture of argon ( $f_{Ar} = 0.02$ ) was also analyzed. The distributions of the argon concentration  $f_{Ar}$ , density, pressure, temperature, and velocity ratios corresponding to this case with  $Re_2 = 78.5$  and  $K_2 = 0.785$  are presented in Figs. 17a - 17c.

The results of the calculations show that in both cases the gas of the surrounding space does not penetrate through the shock wave into the inner supersonic region of the flow. The property indicated above was noted in experiments of Skovorodko and Chekmarev<sup>20</sup> in the simplest case when the expanding and ambient gases were identical in their molecular properties.

In both cases the continuum approach is not applicable in the flow regions behind the shock waves.

The discussed phenomena and the results of previous studies<sup>21,22</sup> were used for estimating the flow parameters and the jet structure in aerodynamic applications.<sup>22</sup>

#### Concluding Remarks

The Direct Simulation Monte-Carlo and continuum methods used in these studies allow to research effectively the structure of gas-mixture flows and diffusive processes in the gas expansion flows from a spherical source, in the underexpanded jets, and also in the gas compression flows about a sphere in the transitional flow regime between free-molecular and continuum regimes. Good correlation is noticed between testing data and calculated results obtained by the Direct Simulation Monte-Carlo technique.

The diffusive effects are significant for estimation of the shock-wave width, adiabatic temperature, recovery factor, pressure at the stagnation point, the effectiveness of species separation, and ambient gas penetration in the considered typical cases of flow compressing and expanding.

The group of similarity parameters ( $Kn_*$ ,  $Kn_+$ ,  $Kn_*$ ,  $K_2$ ,  $Re_*$ , etc.) was found to identify the flow regime as well as the limits of applicability of the continuum

concepts for studying diffusive effects in low-density gas mixtures.

#### Acknowledgment

The author would like to express gratitude to G. A. Bird for the opportunity of using the DS2G computer program, to V. K. Molodtsov for his participation in developing numerical continuum algorithm for the solving the problem, and also to J. N. Moss for his valuable discussion of the results.

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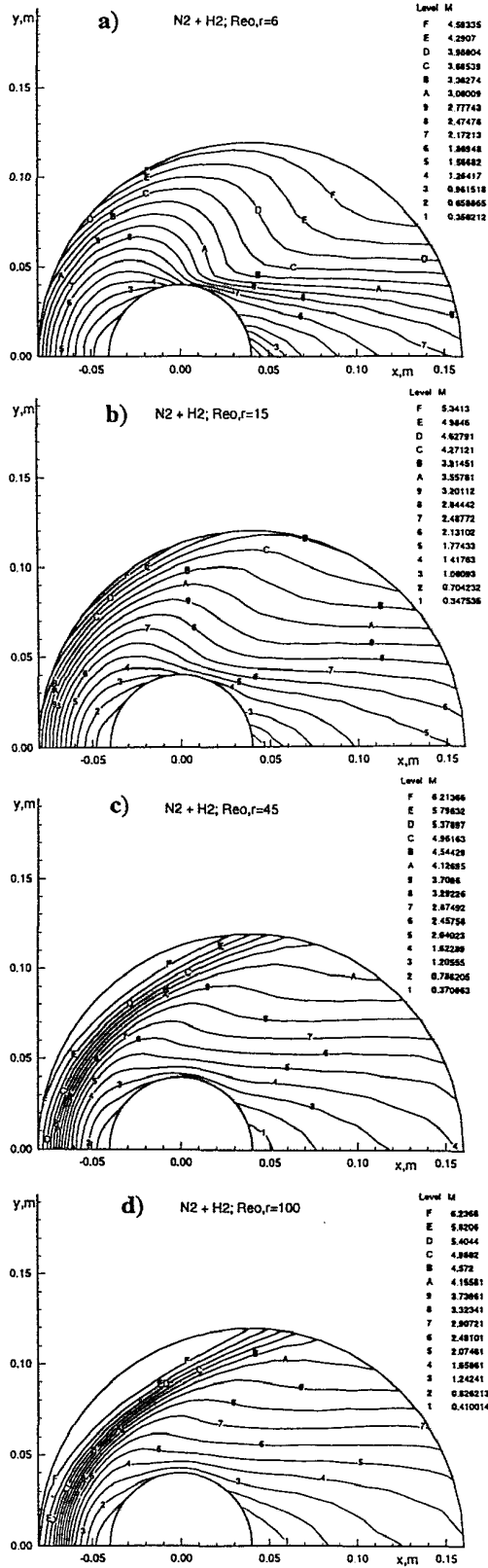


Fig. 1 Mach number contours in the flow of nitrogen-hydrogen mixture at  $M_\infty = 6.6$  past a sphere. The Reynolds number based on the radius of the sphere: a)  $Re_{0,r} = 6$ ; b) 15; c) 45; d) 100.

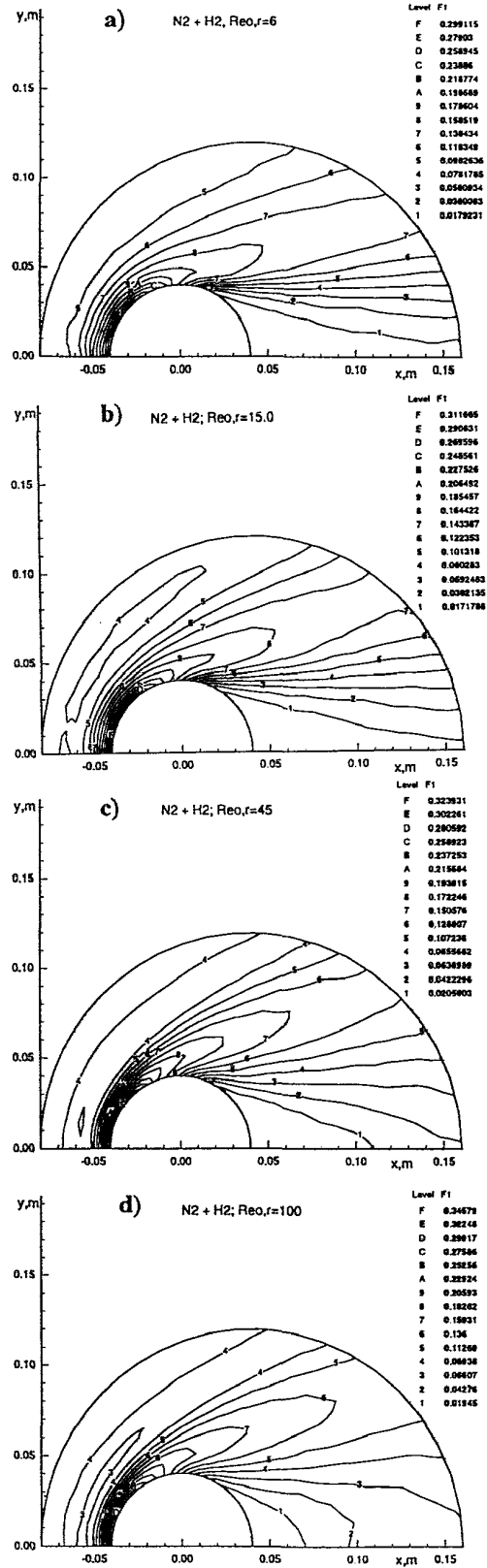
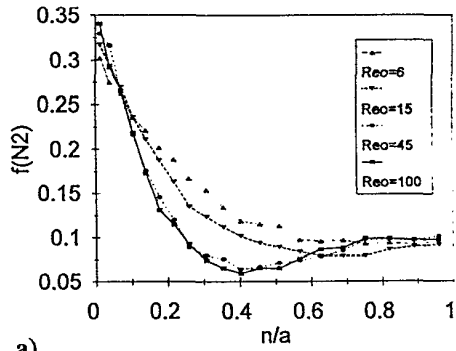
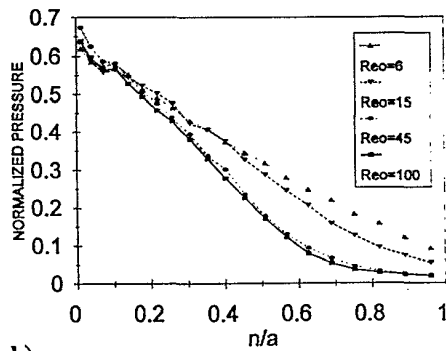


Fig. 2 Nitrogen concentration contours in the flow of nitrogen-hydrogen mixture at  $M_\infty = 6.6$  past a sphere. The Reynolds number based on the radius of the sphere: a)  $Re_{0,r} = 6$ ; b) 15; c) 45; d) 100.



a)



b)

Fig. 3 The profiles of (a) nitrogen concentration  $f_{N_2}$  and (b) normalized pressure  $p/\rho_\infty U_\infty^2$  in the flow of nitrogen-hydrogen mixture at  $M_\infty = 6.6$  near the stagnation line of a sphere.

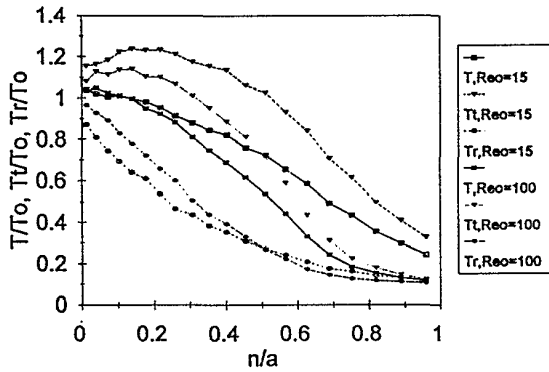
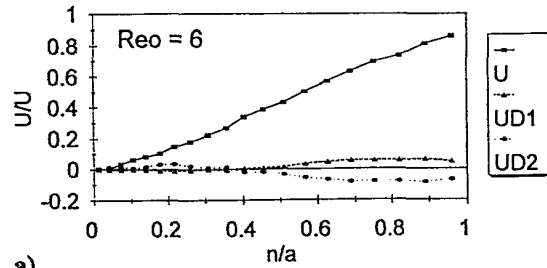
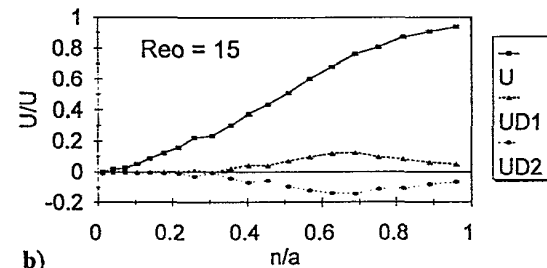


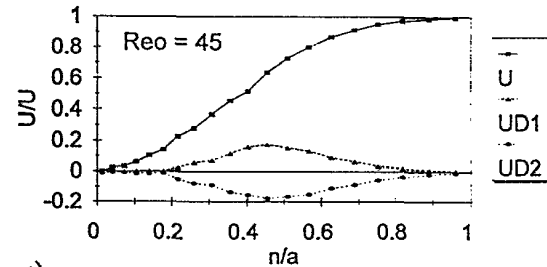
Fig. 4 Distribution of translational ( $T_t$ ), rotational ( $T_r$ ), and overall temperature ( $T$ ) in the flow of nitrogen-hydrogen mixture at  $M_\infty = 6.6$  near the stagnation line of a sphere.



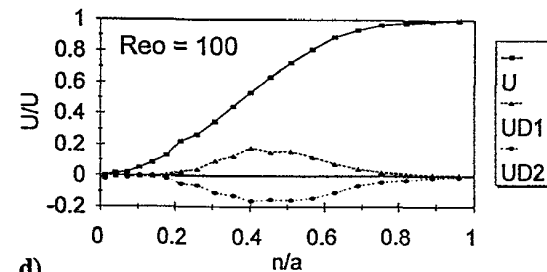
a)



b)



c)



d)

Fig. 5 The profiles of the stream velocity  $U$  and diffusion velocities  $UD1_{N_2}$  and  $UD2_{H_2}$  in the flow of nitrogen-hydrogen mixture at  $M_\infty = 6.6$  near the stagnation line of a sphere.

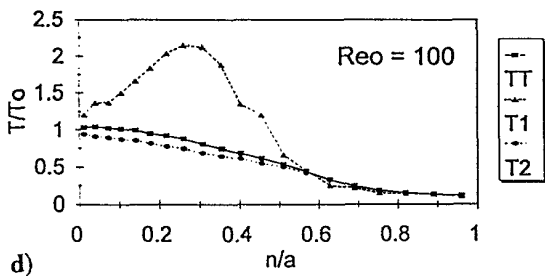
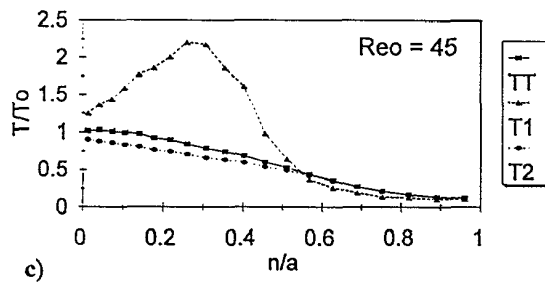
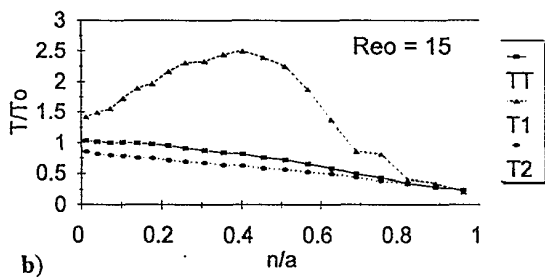
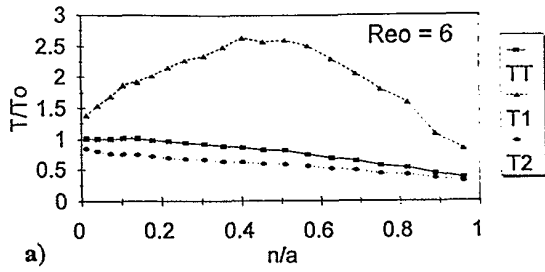


Fig. 6 The profiles of the overall temperature for mixture ( $TT$ ), nitrogen ( $T_1$ ) and hydrogen ( $T_2$ ) in the flow of nitrogen-hydrogen mixture at  $M_\infty = 6.6$  near the stagnation line of a sphere.

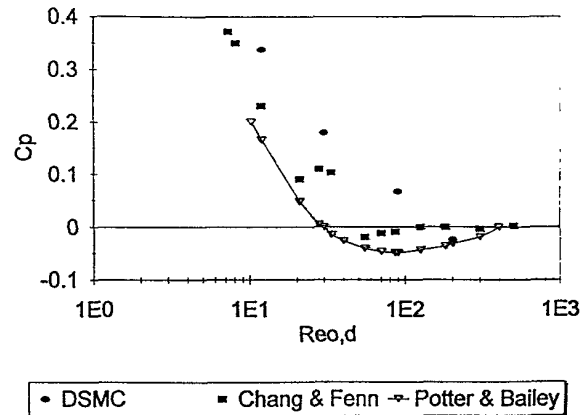


Fig. 7 The pressure coefficient of Homann<sup>34</sup>  $C_p$  as a function of hemisphere-probe Reynolds number  $Re_{o,d}$ . The DSMC results are compared with experimental data of Chang and Fenn<sup>36</sup> and Potter and Bailey.<sup>35</sup>

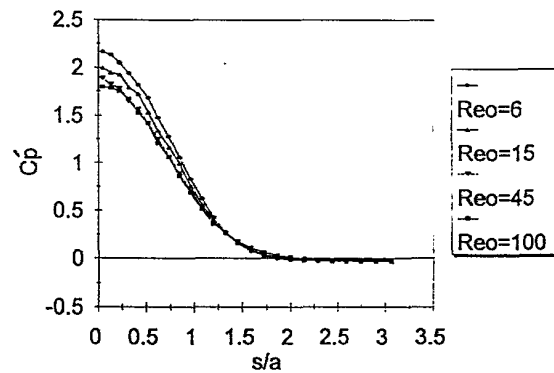


Fig. 8 The pressure coefficient  $C_p'$  along the surface of a sphere.

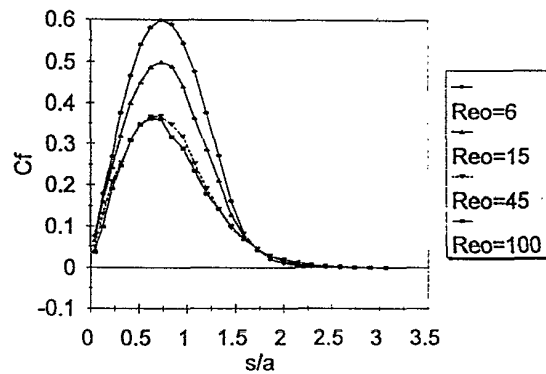


Fig. 9 The friction coefficient  $C_f$  along the surface of a sphere.

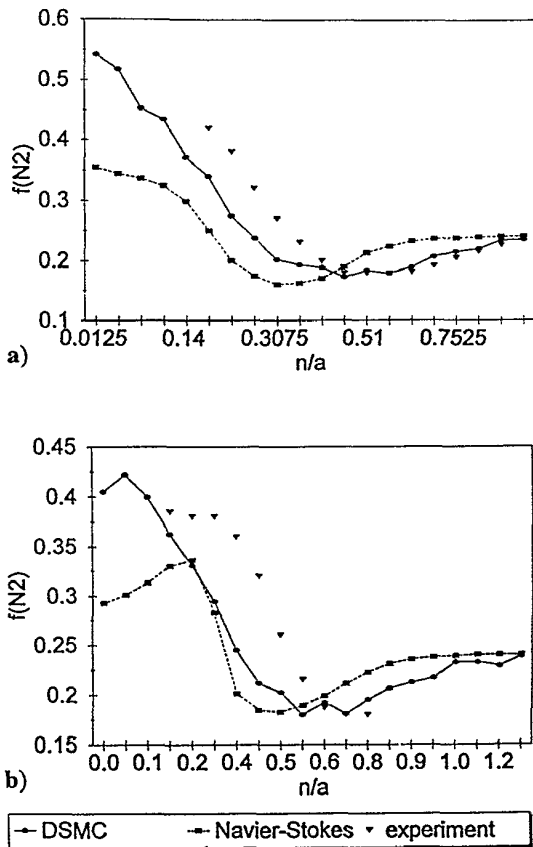


Fig. 10 Nitrogen concentration profiles in the flow of nitrogen-hydrogen mixture at  $M_\infty = 3.5$  and  $Re_{\theta,r} = 51.98$  at (a) the stagnation streamline of a sphere ( $s = 0$ ) and (b) along the normal at  $s = 1.02a$ . The DSMC results are compared with experimental data of Rebrov<sup>1</sup> and Bochkarev and Prikhodko.<sup>16</sup>

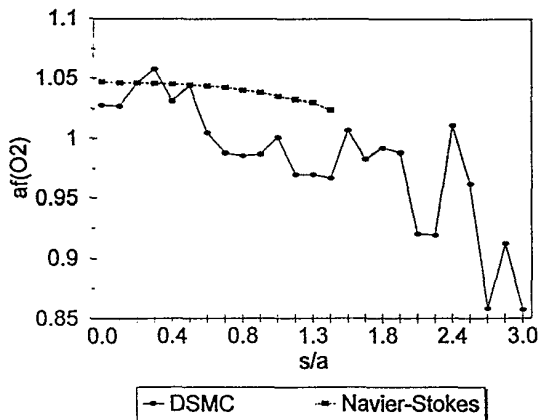


Fig. 13 Species separation effects in an oxygen-nitrogen mixture of air at the surface of a sphere.

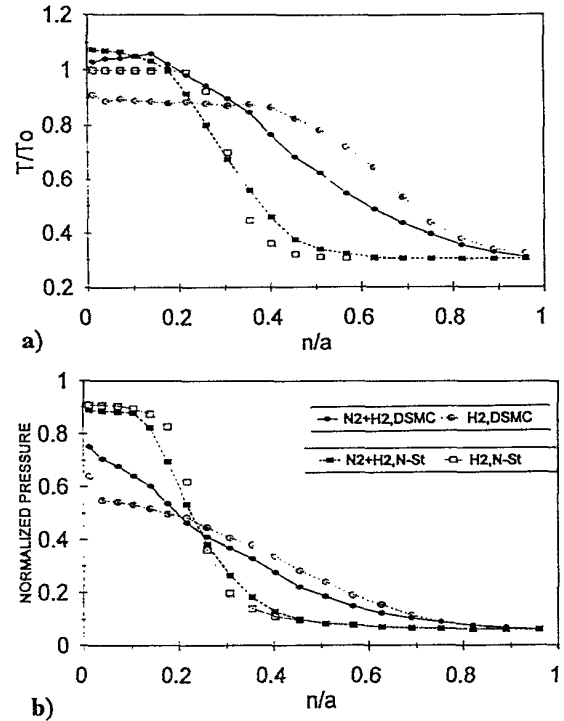


Fig. 11 (a) The overall temperature ( $T$ ) and (b) normalized pressure  $p/\rho_\infty U_\infty^2$  in the flow of nitrogen-hydrogen mixture at  $M_\infty = 3.5$  near the stagnation line

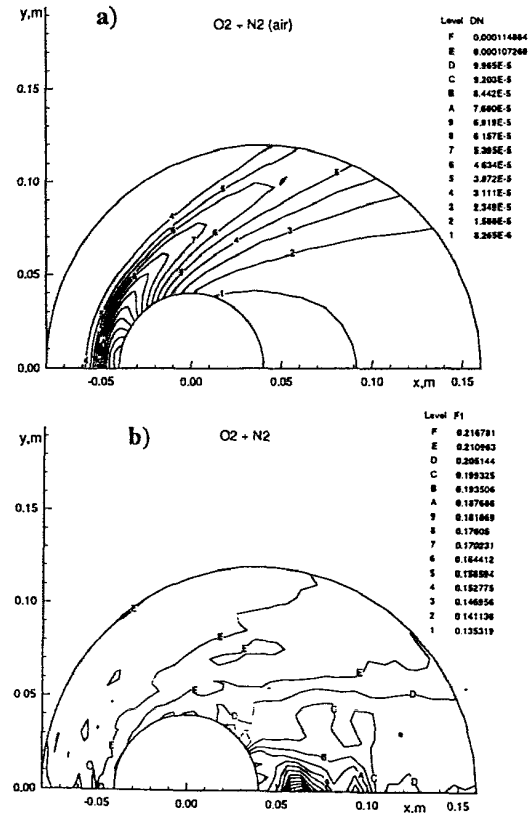


Fig. 12 (a) Density and (b) oxygen concentration contours in the flow of air at  $M_\infty = 3.8$  and  $Re_{\theta,r} = 38.6$  past a sphere.

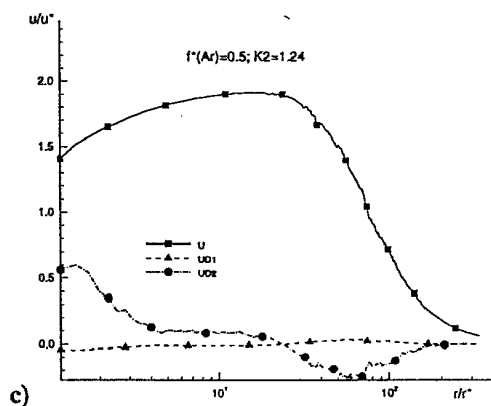
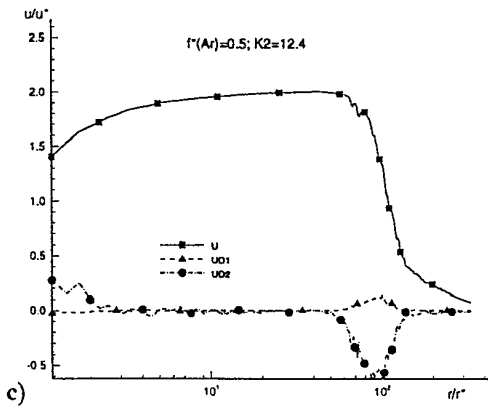
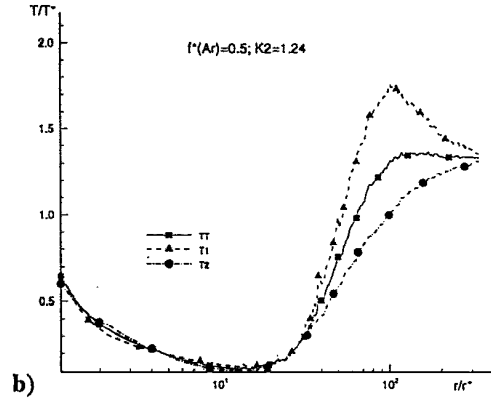
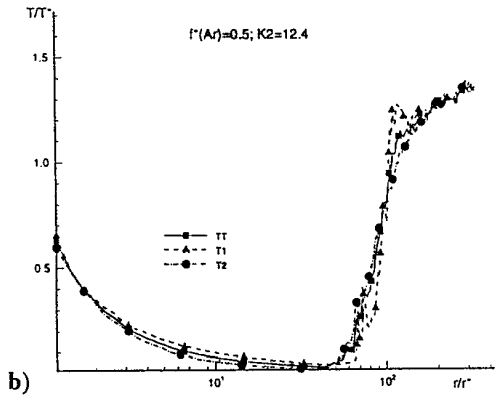
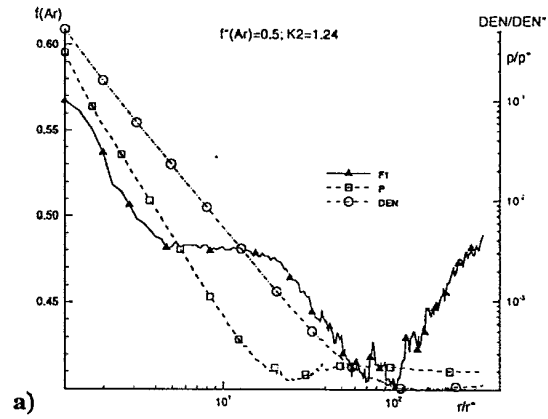
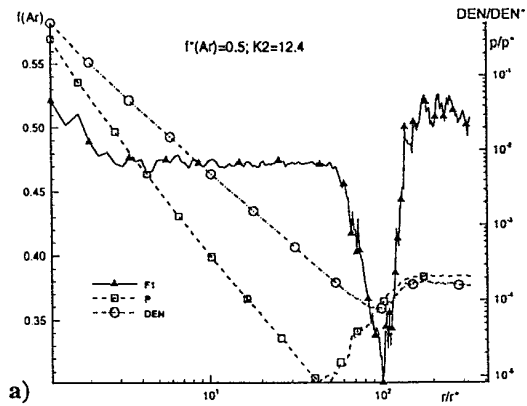


Fig. 14 (a) Argon concentration and density and pressure ratios, (b) temperature ratios, and (c) stream velocity and diffusion velocity ratios in a  $K_2 = 12.4$  expansion of argon-helium mixture.

Fig. 15 (a) Argon concentration and density and pressure ratios, (b) temperature ratios, and (c) stream velocity and diffusion velocity ratios in a  $K_2 = 1.24$  expansion of argon-helium mixture.

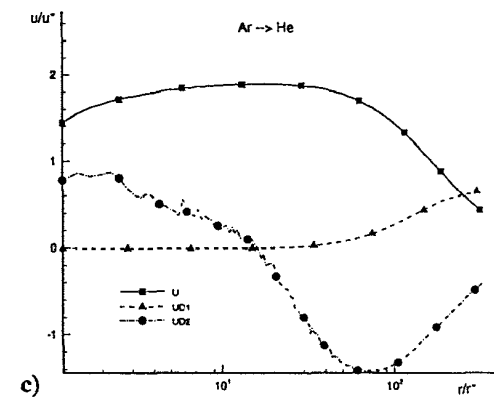
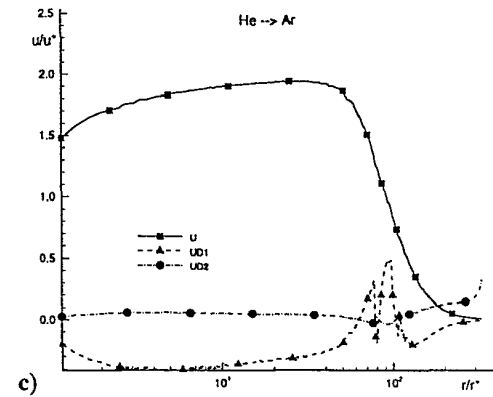
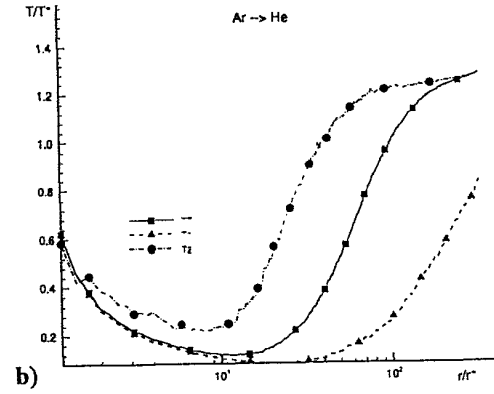
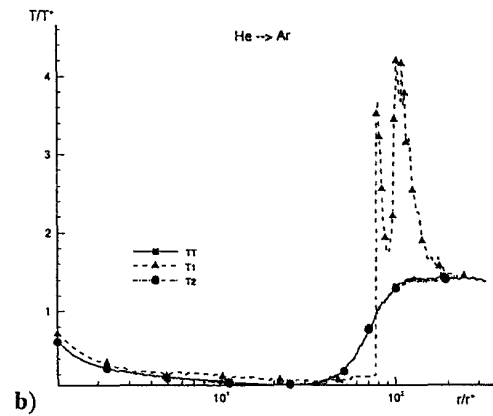
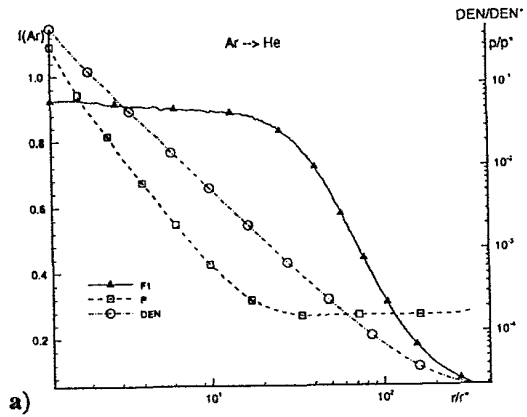
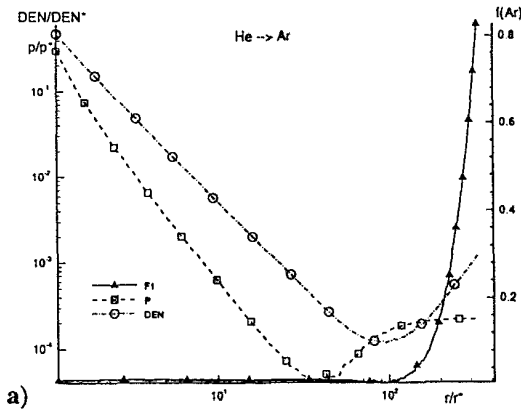


Fig. 16 (a) Argon concentration and density and pressure ratios, (b) temperature ratios, and (c) stream velocity and diffusion velocity ratios in an expansion of helium into argon.

Fig. 17 (a) Argon concentration and density and pressure ratios, (b) temperature ratios, and (c) stream velocity and diffusion velocity ratios in an expansion of argon into helium.