Applications of exponential box schemes for viscous flows with combustion and blowing

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Abstract

A two-point second-order uniform box scheme and effective regularization algorithm have been developed for numerical analysis of the gas flow in the boundary layer (BL) and thin viscous shock layers (TVSL) under conditions of intensive blowing on the body surface and chemical reactions. Flowfield parameters in the boundary layer near the stagnation point of a blunt body were calculated for various blowing factors. Non-equilibrium parameters in the multicomponent TVSL were evaluated for slot and uniform hydrogen injections. Numerical results indicate that the most effective cooling of the probe surface occurs at moderate uniform injections and moderate Reynolds numbers.

Keywords: Exponential box scheme; Regularization algorithm; Gas injection; Supersonic hydrogen combustion; Cooling mechanism

1. Introduction

Different problems of thermophysics and aerodynamics (e.g., stability of supersonic hydrogen combustion and heat protection in hypersonic vehicle engines [1]) come to solving differential equations with small coefficients at the highest derivatives. The latter leads to the formation of regions with small linear dimensions where gradients of functions are large. The numerical analysis of such problems by traditional box schemes [2,3] is limited by non-uniform convergence or even divergence of numerical solutions. In this study, gas flow parameters in boundary and viscous shock layers are analyzed under the conditions of blowing on the body surface and non-equilibrium chemical reactions.

From a mathematical point of view, the increase of the flow rate of blowing gas or chemical-reaction rates is equivalent to the existence of a small coefficient at the highest derivative in the BL equations [3]. A sublayer with large gradients of functions is created. The gas flow in the boundary layer at the body stagnation point is studied using a two-point uniform exponential box scheme. The identical problem was considered in Ref. [4] by using a three-point exponential box scheme. A more effective regularization algorithm [5,6] was used in the present study.

A similar phenomenon is observed in the case of hydrogen combustion at small Reynolds number $Re_0 < 100$ [7,8]. In this study the models of diffusive combustion of hydrogen, which is injected with different intensity from the surface of a parabolic cylinder into airflow, are considered by using the TVSL approach [9] at moderate Reynolds numbers $1500 > Re_0 > 100$.

2. Gas blowing into a boundary layer

Consider the perfect-gas flow in the boundary layer near the stagnation point of a blunt body with uniform blowing at the surface [10]. The system of BL equations acquires the following form [3,5]:

$$U'' + fU' + \beta(S + 1 - U^2) = 0 \tag{1}$$

$$f' = U' \tag{2}$$

$$S'' + \sigma f S' = 0 \tag{3}$$

where the Faulkner–Scan constant [10] $\beta = (1 + j)^{-1}$ characterizes the pressure gradient in inviscid flow; $j = 0$ or 1 in plane and axisymmetric cases correspondingly; and $\sigma = 0.72$ is the Prandtl number.

Boundary conditions are the following.

On the body surface ($Y = 0$) considering the gas blowing:

$$f = f_w = \text{ const}, \ U' = 0, \ S = S_w \tag{4}$$

On the external boundary of the layer ($Y \to \infty$):

$$U = 1, \ S = 0 \tag{5}$$

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In Eq. (4) the parameter f_w characterizes the mass flow rate of the blowing gas. Special box schemes with uniform convergence [11] or exponential schemes [2,4,5] should be used in order to solve the problem at large $|f_w|$. The principal advantages of the two-point box schemes are: (1) any type of boundary conditions "estimated" accurately [5]; (2) algorithmization of the grid-cell changes is simple [6]; and (3) fluxes of the flow parameters are calculated without additional procedure [11], and the approximation error of the fluxes is the same as that of other terms of the equations. The two-point exponential box scheme developed [5] has the second order of uniform convergence. The scheme and its regularization algorithm [5] have been used for the numerical solution of Eqs. (1)–(5) under the conditions of moderate and intensive blowing from the thermally isolated body surface ($S_w = 0$). The profiles of the tangential component of the velocity U and its derivative U' along the normal at the stagnation point on the surface of the axisymmetrical blunt body ($\beta = 0.5$) are shown in Figs. 1 and 2 for various blowing parameters ($f_w = 0,\ -2.5,\ -10,\ -25$). The presence of the blowing flow significantly changes the flow structure. As the rate of blowing increases, the boundary layer becomes thicker, and the friction on the body surface decreases.

3. Combustion in the thin viscous shock layer

The analysis of hypersonic hydrogen combustion processes in the viscous shock layer near a parabolic cylinder is based on the TVSL equations and boundary conditions [8]. Mathematical description of the non-equilibrium process of hydrogen combustion considered 11 gas components (O_2, N_2, NO, H_2O_2, HO_2, H_2O, OH, H_2, H, O, N) and 35 chemical reactions [12]. An approximation to the equations was constructed using a matrix variant of two-point exponential box scheme [5,6,11]. Modified Newton–Raphson method [8] was used for solving the nonlinear grid equations.

3.1. Slot injection

The slot injection from the body surface was simulated by using the injection parameter $G = G_0 \exp(-\alpha_w / X)$, where G_0 is the value of the parameter at the critical cylinder line, $\alpha_w = 30$, and X is marching coordinate along the surface. The computational results indicate the presence of significant amounts of nonreacting molecular hydrogen near the body. For $Re_0 > 100$, the diffusion of the "hot" reaction products (water and OH) occurs from the reaction zone to the surface. This phenomenon leads to the increase of heat flux values at the surface.

Figure 3 presents distributions of the heat flux $Q = 2q/(\rho_\infty V_\infty^2)$ towards the body surface along the coordinate X. The heat from combustion reactions does not prevail over the cooling effect of injection at the considered Reynolds number, $Re_0 = 628$. Even at a low level of injection, the local heat flux decreases (see squares in Fig. 3). The heat flux is significantly lower than Q at $G_0 = 0$ near the injection zone at moderate injections ($0.02 \leqslant G_0 \leqslant 0.05$). A large quantity of injected hydrogen leads to effective cooling of the surface at a large distance from the injection zone at a high level of injection (see diamonds in Fig. 3). The combustion zone moves from the

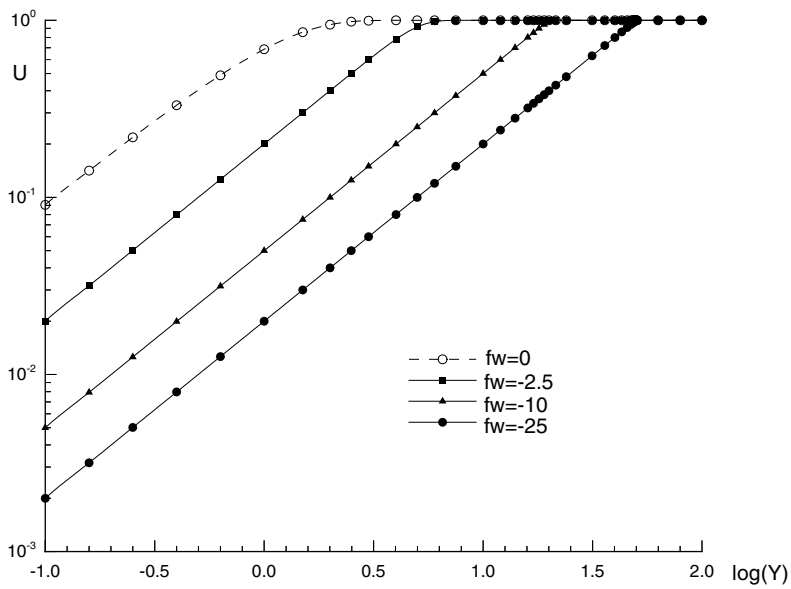

Fig. 1. Function U across the boundary layer for various blowing factors.

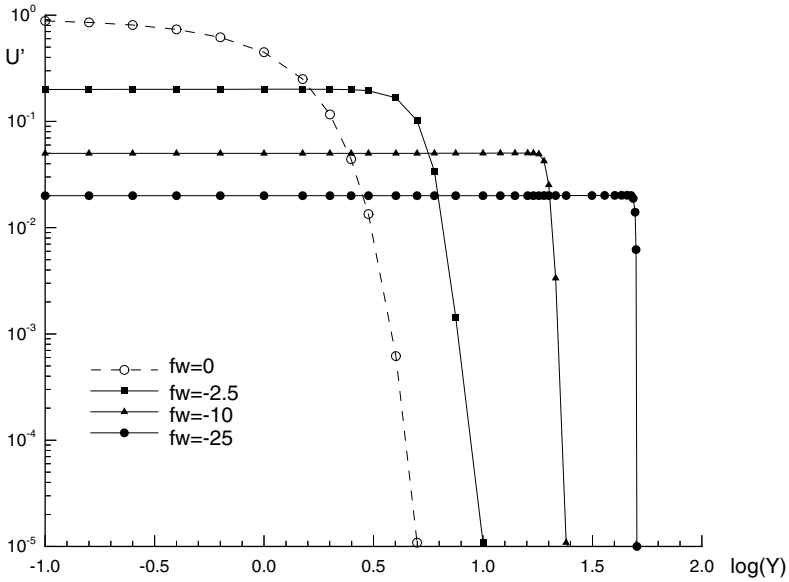

Fig. 2. Function U' across the boundary layer for various blowing factors.

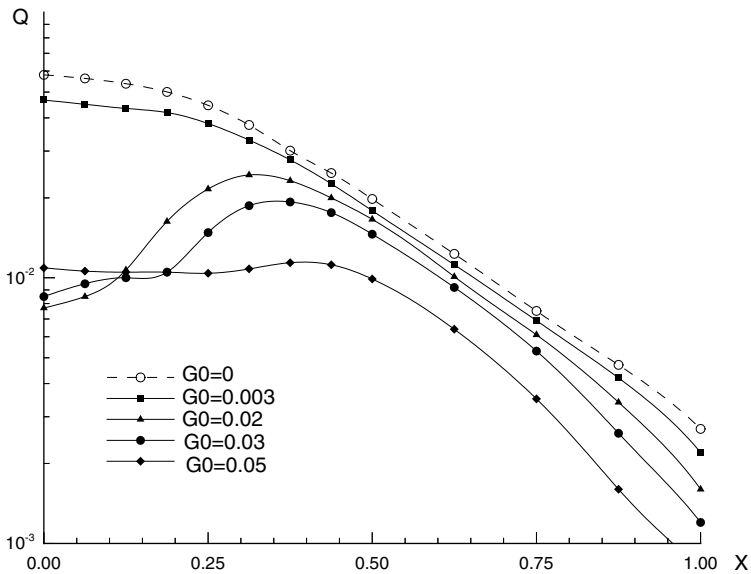

Fig. 3. Heat flux on parabolic cylinder at slot injections.

surface towards the external boundary of the TVSL as the injection intensity increases.

3.2. Uniform injection

Cooling the porous surface of the parabolic cylinder was modeled at the constant intensity of hydrogen injection, $G = G_0 =$ const along the coordinate X. More significant decrease of the heat flux is found at the whole body surface, compared to the case of slot injection at $Re_0 = 628$. For the weak injections (squares in Fig. 4), the heat flux at the entire computational region is less than Q in the absence of the injection, i.e. $G_0 = 0$. The distribution of the heat flux becomes noticeably distinct at moderate (triangles, $G_0 = 0.02$) and strong (circles, $G_0 = 0.03$) injections, as the result of mutual influence of thermophysical properties of hydrogen, high values of enthalpy of upstream flow, and the presence of combustion zone in the viscous shock layer.

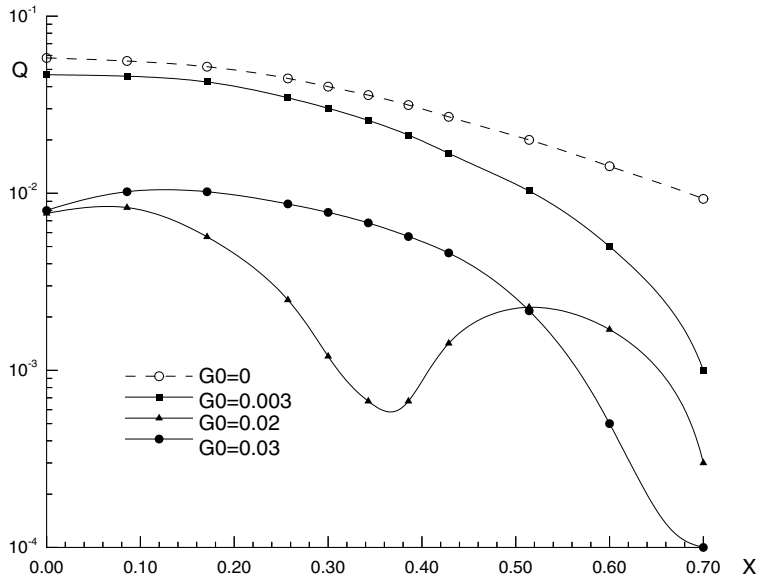

Fig. 4. Heat flux on parabolic cylinder at uniform injections.

4. Conclusions

The present results demonstrate that the moderate blowing leads to decreasing the friction on the surface. The flow parameters in this case should be calculated using the two-point exponential box scheme that has a property of uniform second-order convergence in the full range of blowing parameters.

The other goal was the development of a numerical tool for studying the processes of diffusive hydrogen combustion in a thin viscous shock layer at moderate Reynolds numbers. The algorithm combines a matrix factorization variant of the two-point exponential box scheme and the Newton–Raphson method for solving the nonlinear grid equations. The numerical results indicate that the major reaction products are water and *OH*, which could be used for the identification of the excess/deficient hydrogen zones. The most effective cooling of the surface occurs at moderate uniform hydrogen injection.

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