Limitations of the bulk viscosity approach in modeling the expanding nitrogen flows

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Limitations of the Bulk Viscosity Approach in Modeling the Expanding Nitrogen Flows

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Abstract. The accuracy of application of the bulk viscosity approach in modeling the spherically-expanding flows of nitrogen has been evaluated in comparisons of distributions of translational and rotational energy calculated within the framework of the Chapman-Enskog method and solutions of the nonequilibrium gasdynamic equations and the relaxation equation. The calculations of the bulk viscosity, relaxation time, shear viscosity, thermal conductivity, and diffusion coefficients are carried out in the temperature range from 10 K to 1,000 K for nitrogen using the classical Parker's model (at moderate temperatures) and the quantum-mechanics adiabatic energy-exchange approach (at temperatures $T < 100$ K). The numerical solutions of the Navier-Stokes equations are analyzed for both approaches under the conditions of nitrogen flow in underexpanded jets. It is found that the bulk viscosity approach predicts much thinner spherical shock-wave areas than those estimated by the relaxation-equation approximation. The use of large values of the bulk viscosity ratio (estimated using the quantum-mechanical model) in calculations leads to distributions of rotational temperature along the radial ray that do not have any physical meaning and do not match any known experimental data for expanding nitrogen flows.

INTRODUCTION

The correlation of the non-equilibrium gas-flow model (based on the rotational relaxation equation) and the bulk-viscosity approach for the plane shock was studied in [1]. In this paper, the similar analysis is made for spherically-expanding nitrogen flows.

A general theory of bulk viscosity, based on the generalized Chapman-Enskog method, and problems associated with the definition of temperature were discussed in [2]. The bulk viscosity $\text{æ}=4/25 \cdot p\tau_R$ appears in the viscous-stress tensor, if temperature (T) is defined as a measure of the total energy of molecules. If T is known (e.g., from the solution of the Navier-Stokes equations with the æ term), then the translational temperature (T_t) and the rotational temperature (T_R) can be found from the following equations [1, 2]:

$$T_t = T - (\text{æ}/nk)\cdot\text{div}(\bar{u}) \tag{1}$$

$$T_R = 5/2\cdot T - 3/2\cdot T_t \tag{2}$$

Here $\bar{u}$ is the velocity vector, n is numerical density, and k is the Boltzmann's constant.

The bulk-viscosity approach is valid only under two criterial conditions:

$$\tau_t/\tau_R \leq 1 \tag{3}$$

$$\tau_R/\theta \ll 1 \tag{4}$$

where τ_t is the translational-relation time and θ is the gasdynamic time. The first condition (see Eq. 3) is satisfied in various applications, but the second one (see Eq. 4) refers to the *near-equilibrium* states (e.g., in the cases of absorption and dispersion of sound [2]).

In the "relaxation" approach [3], T_t and T_R are defined as measures of kinetic and rotational energies of molecules; the term with the bulk viscosity is omitted, and the τ_R-relaxation equation is added to the system of Navier-Stokes equations [2, 3].
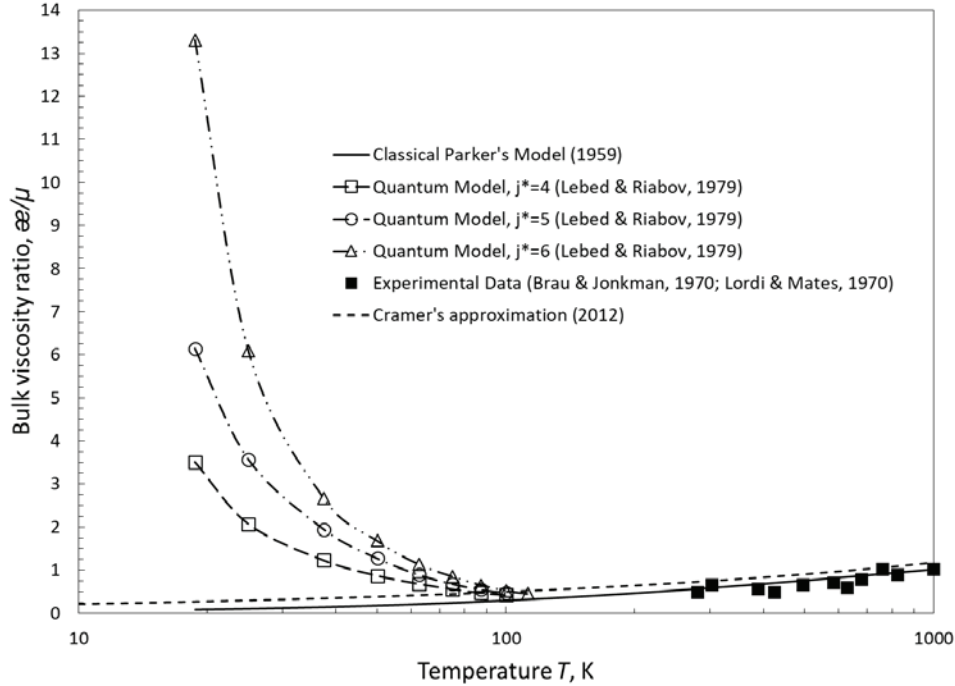
ESTIMATING THE VALUES OF THE BULK VISCOSITY RATIO

FIGURE 1. The values of the bulk viscosity ratio, ϖ/μ at various values of temperature T. Classical Parker's model predictions from [2, 4, 5]; Cramer's approximation from [6]; experimental data from [7, 8]; quantum mechanics estimations from [13].

The estimations of the bulk viscosity ratio, ϖ/μ have been made in numerous studies (see Fig. 1):
- Parker [4] carried out a classical analysis of deriving the expression for the rotational relaxation times of diatomic gases. The revised Parker's formula was received in [5]. This data is shown as a solid line in Fig. 1.
- Cramer's approximation formula [Ref. 6, pp. 7-8] has also been used (see the dashed line in Fig. 1).
- Experimental data of Brau and Jonkman [7] and Lordi and Mates [8] is shown as filled-square markers in Fig. 1.
- The values of bulk viscosity of dilute gases (including N_2) were also estimated in [9-11]. They correlated well with data from [4-8]. Sherman [11] made the assumption that $\varpi/\mu = 2/3$ to quantitatively match normal shock-wave profiles and shock widths in a low-density wind tunnel for air at supersonic Mach numbers from 1.78 to 3.91.
- Quantum effects [12, 13] may play a significant role in expanding nitrogen flows. The results [13] are shown in Fig. 1 for different rotational levels: $j^* = 4$, 5, and 6 (empty squares, circles, and triangles, correspondingly).

NUMERICAL METHOD

In this paper, the combined effect of rotational-translational relaxation and dissipative processes of viscosity and conductivity in expanding spherical flows of nitrogen is studied. The system of the Navier-Stokes equations and the relaxation equation, based on the τ_R-approximation [3], was solved by the implicit numerical technique [14]. Solutions of the boundary value problem depend on the following parameters: Reynolds number $Re_* = \rho_* u_* r_* / \mu(T_*)$; pressure ratio $P = p_*/p_a$; stagnation temperature T_0; viscosity ($\mu \sim T^n$) and relaxation-time approximation ($p\tau_R \sim T^m$) parameters

n and m; and the relaxation similarity parameter $K_* = p\tau_R(T_*)/\mu(T_*)$. Here parameters marked with the index $(_*)$ are calculated at the sonic conditions.

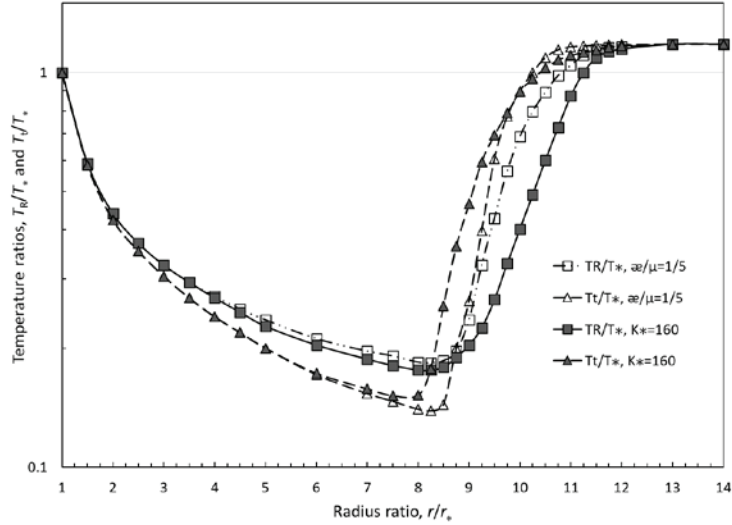## NUMERICAL RESULTS: COMPARING TWO APPROACHES

FIGURE 2. T_R/T_* rotational and T_t/T_* translational temperature ratios in the spherical expanding flow of nitrogen at the relaxation similarity parameter $K_* = 160$ and the bulk-viscosity ratio $æ/\mu = 1/5$.

FIGURE 3. T_R/T_* rotational and T_t/T_* translational temperature ratios in the spherical expanding flow of nitrogen at the relaxation similarity parameter $K_* = 48$ and the bulk-viscosity ratio $æ/\mu = 2/3$.
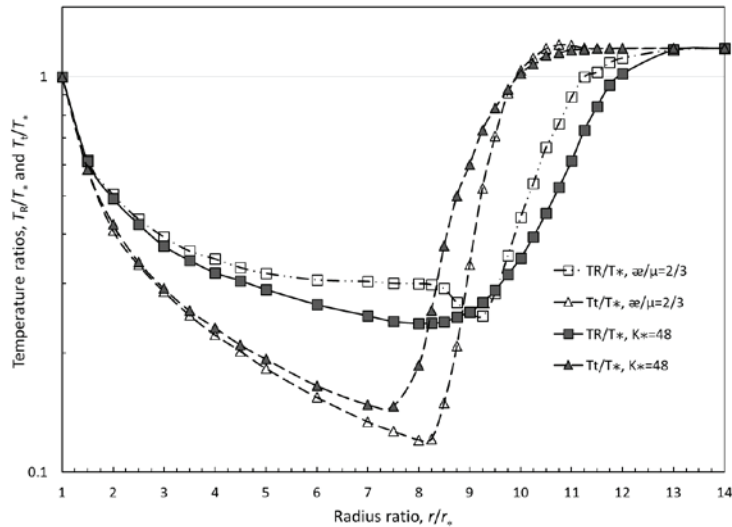
Numerical results of the rotational temperature T_R (squares) and translational temperature T_t (triangles) distributions along the radial direction in the spherically-expanding nitrogen flows at the parameters $Re_* = 200$, ρ_∞/ρ_* = 0.02, $Pr = 0.7$, and $m = n = 1$ are shown in Figs. 2 and 3 for two values of the similarity criterion $K_* = 160$ and 48 (solutions of Navier-Stokes equations and τ-relaxation equations; see the filled markers), and for the corresponding values of bulk viscosity ratios $æ/\mu = 1/5$ and 2/3 (solutions of Navier-Stokes equations with the bulk-viscosity term; see the empty markers). The current results correlate well with the numerical solutions from [15] at $æ/\mu = 2/3$.

The decrease rate for T_R slows down with the gas expanding in the inner supersonic area of the flow that leads to the "frozen" value of rotational temperature T_R. The decrease of the relaxation parameter K_* leads to a higher "frozen" value of T_R there. Rotational-translational equilibrium never exists near the shock-wave front in such flows, and $T_R > T_t$. The further sharp decrease of T_R values calculated with the bulk-viscosity approach at $\mathit{æ}/\mu = 2/3$ (see Fig. 3, empty squares) does not have a clear physical meaning, and it is likely that this approach does not work in this area of the spherical shock wave. Also, it is found that the $\mathit{æ}$–approach predicts much thinner spherical shock-wave areas than those estimated by the τ_R–relaxation approximation (see Figs. 2, 3). Therefore, in the cases of the significant "delay" values (that take places in the expanding diatomic-gas flows [3, 16]), it is more preferable to define the temperature by the kinetic energy of molecules (T_t) and use the relaxation equation for T_R with the τ_R–approximation, omitting $\mathit{æ}$ in the gasdynamic equations.

The solutions of the Navier-Stokes equations and the τ_R–relaxation equation using the quantum energy-exchange model [13] correlate well with the experimental data [12] on the T_R rotational-temperature distributions along the axis of the underexpanded rarefied-nitrogen jet flow, which can be approximated by the spherical flow there [16]. Unfortunately, the use of large values of the bulk viscosity ratio ($\mathit{æ}/\mu > 1$) in the $\mathit{æ}$–approach estimates leads to distributions of rotational temperature T_R along the radial ray that do not have any physical meaning and do not match any known experimental data for expanding nitrogen flows.

CONCLUSIONS

At temperatures $T > 100$ K, the values of bulk viscosity of nitrogen were estimated following the studies from Refs. [4-11]. In numerical calculations, we used the bulk-viscosity ratio $\mathit{æ}/\mu = 2/3$ that was recommended by Sherman [11] to quantitatively match normal shock profiles and shock widths in a low-density wind tunnel for air at supersonic Mach numbers. It is found that the $\mathit{æ}$–approach predicts much thinner spherical shock-wave areas than those estimated by the τ_R–relaxation approximation.

This study shows that, in the cases of the significant internal-energy-exchange "delay" values (that take places in the expanding diatomic-gas flows [3, 16]), it is more preferable to define the temperature by the kinetic energy of molecules (T_t) and use the relaxation equation for the rotational temperature T_R (defined as a measure of the rotational energy of molecules) with the τ_R–approximation [3, 14, 17], omitting the bulk-viscosity term $\mathit{æ}$ in the gasdynamic equations.

The use of large values of the bulk viscosity ratio ($\mathit{æ}/\mu > 1$, estimated using the quantum-mechanical model [13] at $T < 100$ K) in the $\mathit{æ}$–approach calculations leads to distributions of rotational temperature T_R along the radial ray that do not have any physical meaning and do not match any known experimental data for expanding nitrogen flows.

ACKNOWLEDGMENTS

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