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Aerodynamic Applications of Underexpanded Hypersonic Viscous Jets

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Nomenclature

A = constant, Eq. (7)
C = constant, Eq. (10)
c_f = friction coefficient
c_x = drag coefficient
c_y = lift coefficient
E_R = rotational energy
G = transformed temperature function
f = concentration of heavy component
h = plate thickness
I = Bessel function
J = Bessel function
K = hypersonic similarity parameter, $M_\infty \sin \alpha$
K_2 = similarity parameter, $Re_*(p_a/p_s)^{1/2}$
K_* = relaxation parameter, $\rho_* u_* r_*/(p\tau_R)_*$
l = length of model
M = Mach number
M_i = molecular weight of i species
m = relaxation parameter, Eq. (10)
n = exponent in the viscosity law $\mu \sim T^n$
$\langle P_{j^*,j^*-2} \rangle$ = average transition probability per one gas-kinetic collision for j^* rotational level
Pr = Prandtl number
p = pressure
q = dynamic pressure, $\frac{1}{2}\rho_\infty u_\infty^2$
R = asymptotic parameter, $0.75Re_{*i}$
Re_j = Reynolds number calculated by flow parameters in the nozzle exit cross section, $\rho_* u_* r_j/\mu_j$
Re_0 = Reynolds number and the main similarity parameter, $\rho_\infty u_\infty l/\mu_\infty$
Re_* = Reynolds number calculated by flow parameters at the critical sphere, $\rho_* u_* r_*/\mu_*$
r = distance from a nozzle exit along the ray
r_d = location of the shock wave ("Mach disk") on the jet axis
r_* = radius of spherical source
Sc = Schmidt number
s = exponent in the molecular interaction law
T = temperature
t = dimensionless temperature function, T/T_{*i}
t_w = temperature factor, T_w/T_s
u = radial velocity component
V = transformed velocity function, Eqs. (4) and (6)
v' = tangential velocity component, Eq. (1)
v = dimensionless velocity, u/u_{*i}, Eqs. (6) and (8)
W = transformed velocity function, Eqs. (4) and (8)
X = transformed coordinate, Eqs. (4) and (6)
X = drag force
x = inverse dimensionless radius, r_{*i}/r
Y = lift force
Z = transformed coordinate, Eq. (8)
α = angle of attack
β = thermal diffusion ratio
γ = ratio of specific heats
δ = dimensionless plate thickness, h/l
ε = ratio of molecular weights, M_{He}/M_{Ar}
η = argument for Bessel functions, Eq. (7)
Θ = transformed temperature function, Eq. (8)
θ = angle between the body generatrix and the freestream flow
λ = expansion parameter, $2\omega(\gamma - 1)$, Eq. (4)
μ = viscosity coefficient
ξ = deformable coordinate, Eq. (4)
ρ = density
τ = relaxation time
Φ = transformed concentration function, Eq. (6)
φ = angle between the ray from a nozzle exit and symmetry axis
Ψ = transformed concentration function, Eq. (8)
ω = expansion parameter, $1/[2\gamma - 1 - 2(\gamma - 1)n]$, Eq. (4)

Subscripts

a = ambient media parameter
d = "Mach disk" parameter
FM = free-molecular-flow parameter
i = inviscid gas parameter
j = nozzle exit parameter
j^* = rotational quantum level
R = rotational relaxation parameter
s = stagnation parameter
t = translational temperature parameter
w = wall quantity
1 = first-order approximation
∞ = freestream quantity
$+$ = coordinate, at which gas density is extreme
$*$ = critical value parameter

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$*i$ = quantity on the critical surface of the ideal gas source

Superscripts

$^\circ$ = equilibrium state

$'$ = dimensionless quantity relative to the parameters under nozzle exit conditions

Introduction

UNDEREXPANDED viscous jets have become widely used to study nonequilibrium thermo- and gasdynamic processes in hypersonic rarefied gas flows[1-4] and aerothermodynamic characteristics of hypersonic vehicle models in wind tunnels.[5,6] The purpose of the present study is to analyze transonic and hypersonic regions of underexpanded viscous jets, their diffusion and rotational-translational nonequilibrium processes, as well as to apply the jet theory to hypersonic aerodynamic research. Using the method of deformable coordinates, the asymptotic solutions are found for jet parameters in transonic and hypersonic regions. Rotational-translational relaxation is analyzed by the numerical solutions of the Navier-Stokes equations in terms of classical and quantum concepts. The aerodynamic characteristics of wedges and plates were investigated in rarefied jet flows of helium, argon, nitrogen, air, and carbonic acid. Fundamental laws for the characteristics and similarity parameters are discussed.

Inviscid Gas Jets

Consider the steady expansion of a jet from an axisymmetric nozzle of exit radius r_j. For a inviscid gas flow in a hypersonic region the following asymptotic solution[7,8] of the Euler equations was found:

$$u' = u_0 + u_1\left[\frac{r'_*(\varphi)}{r'}\right]^{2(\gamma-1)} + \cdots$$

$$v' = v_1\left[\frac{r'_*(\varphi)}{r'}\right]^{2(\gamma-1)}\frac{d \ln r'_*(\varphi)}{d\varphi} + \cdots$$

$$\rho' = \frac{1}{u_0}\left[\frac{r'_*(\varphi)}{r'}\right]^{2} + \cdots$$

$$T' = \theta_1\left[\frac{r'_*(\varphi)}{r'}\right]^{2(\gamma-1)} + \cdots \tag{1}$$

$$p' = \frac{\theta_1}{u_0}\left[\frac{r'_*(\varphi)}{r'}\right]^{2\gamma} + \cdots$$

$$u_0 = \sqrt{\frac{\gamma+1}{\gamma-1}}, \quad u_1 = -\frac{\theta_1}{\sqrt{\gamma^2-1}}, \quad r'_*(\varphi) = \frac{r_*(\varphi)}{r_j}$$

$$v_1 = -\frac{2\theta_1}{u_0[1-2(\gamma-1)]}, \quad \theta_1 = \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2(\gamma-1)}$$

where $r'_*(\varphi)$ is some arbitrary function of angle φ, and the dependent variables are made dimensionless relative to their critical values at Mach number $M = 1$. Gusev et al.[7,8] have found that at large distances from the nozzle exit the isentropic flow in a jet asymptotically approaches, along each ray φ = const, the flow from some spherical source having an intensity that varies from ray to ray. The results of numerical calculations by the method of characteristics are usually used to obtain the function $r_*(\varphi)$.[7]

Using Eq. (1) we can find the asymptotic solution for Mach number M along the axis of an axisymmetric nonviscous gas jet at large distances $r' \gg 1$:

$$M = \left(\frac{\gamma+1}{\gamma-1}\right)^{(\gamma+1)/4}\left[\frac{r'}{r'_*(0)}\right]^{\gamma-1} \tag{2}$$

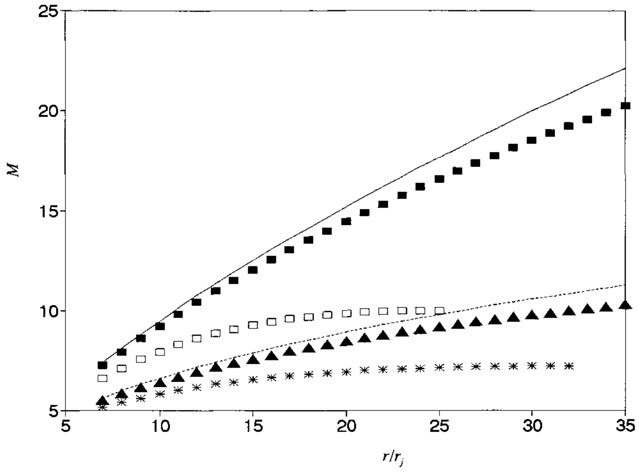


Fig. 1 Mach number M along the axis of axisymmetric jet: solid line, inviscid gas, $\gamma = 5/3$; ■, Ar, $Re_j = 800$; □, He, $Re_j = 200$; dashed line, inviscid gas, $\gamma = 1.4$; ▲, N_2, $Re_j = 800$; *, H_2, $Re_j = 200$.

The result of calculations of M is presented in Fig. 1 for monatomic (solid line, $\gamma = 5/3$) and diatomic (dashed line, $\gamma = 1.4$) gases.

The structure of inviscid gas jets was analyzed by Ashkenas and Sherman,[1] Muntz,[2] and Gusev and Klimova[7] in detail. The main feature of the jet flow is that the flow inside the jet bounded by shock waves becomes significantly overexpanded relative to the outside pressure p_a. This degree of overexpansion has a maximum value. If sonic Mach number $M_j = 1$ in the initial cross section of the jet, and if the pressure $p_j \gg p_a$, this value is determined by the location of the front shock wave ("Mach disk") on the jet axis r_d[1]:

$$r_d/r_j = 1.34(p_s/p_a)^{1/2} \tag{3}$$

The overexpansion phenomenon is very important for aerodynamic experiments in vacuum wind tunnels. In this case the restoration of the pressure occurs automatically, without the use of a diffuser.

Viscous Gas Jets

The analysis of strongly underexpanded jets of viscous gas was performed by Gusev et al.,[5,8] and Gusev and Mikhailov.[9] It was shown that at distances $r' = \mathcal{O}(Re_j^\omega)$, from the source in the flow, dissipative processes become important and the asymptotic solutions [see Eq. (1)] are not applicable in this region. To find the solution here we use the Navier-Stokes equations in a spherical coordinate system[8] with the origin in the nozzle exit cross section. These new independent and dependent variables should be introduced:

$$W = \frac{u'-u_0}{\xi^A}, \quad V = \frac{v'}{\xi^A}, \quad \gamma = 2\omega(\gamma-1)$$
$$\Theta = \frac{T'}{\xi^A}, \quad X = \frac{r'_*(0)}{r'\xi^\omega}, \quad \xi = \frac{4}{3Re_j r'_*(0)} \tag{4}$$

After the change and the transition to $\xi \to 0$, we obtain the solution of the Navier-Stokes equations[8] at $n \neq 1$:

$$W = -\frac{\Theta}{u_0(\gamma-1)} - \frac{u_0\Theta^n}{r_0^2 X}, \quad r_0 = \frac{r'_*(\varphi)}{r'_*(0)}$$

$$\Theta = \left[\frac{\gamma(\gamma+1)(1-n)\omega}{r_0^2 X} + \theta_1^{1-n}(r_0 X)^{2(\gamma-1)(1-n)}\right]^{1/(1-n)} \tag{5}$$

$$r_0^2\left(\frac{\partial V}{\partial X} - \frac{V}{X}\right) - \frac{1}{\gamma u_0 X}\frac{\partial}{\partial\varphi}(r_0^2\Theta) + \frac{n u_0 \Theta^{n-1}}{2X^2}\frac{\partial\Theta}{\partial\varphi} = 0$$

This solution correlates with the solution of Freeman and Kumar[10] at $\varphi = 0$. Therefore, the viscous flow along the axis of an axisymmetric jet at large distances from the nozzle exit asymptotically approaches the one-dimensional flow from a spherical source. Parameter $r'_*(0)$ could be approximated by functions[7] of M_j and γ.

The influence of viscosity on the gas flow near the jet axis was analyzed by Gusev et al.[8,11,12] in terms of the one-dimensional analogy. Also they found a good correlation of some experimental data with the similarity law,[9] such as for the coordinate $r_+ \sim r_d$, at which the density is extreme, as well for density ρ_+. The influence of dissipative viscous effects on velocity and density along the jet axis is insignificant.

The results of calculations of M along the axis of the axisymmetric jet of a viscous gas are presented in Fig. 1 for argon ($Re_j = 800$), helium ($Re_j = 200$), nitrogen ($Re_j = 800$), and hydrogen ($Re_j = 200$) at a stagnation temperature $T_s = 295$ K. For He and H_2 the calculations were done under the condition $\Theta \geq \Theta_+$.

The present analysis demonstrates the equivalence of the flow in the hypersonic region of a jet to one-dimensional spherical flow, which was also studied by Ladyzhenskii.[13] Therefore, we continue the analysis of rotational nonequilibrium flows and binary mixture flows based on the model of the spherical gas expansion.

Spherical Expansion of a Binary Gas Mixture

The multicomponent gas jet flows could be used for simulating hypersonic vehicle flight in Earth and Mars atmospheres as well as for the separation of isotopes or gas mixtures. Consider the spherical expansion of a binary gas mixture into a flooded space as a theoretical model for the study of separation processes in the axisymmetric jets.

Following the study of Chapman and Cowling,[14] the system of one-dimensional Navier-Stokes equations and boundary conditions were written for a one-temperature gas mixture in the case of spherical symmetry.[15] The system of equations was solved numerically by the method developed by Gusev and Zhbakova.[11]

The solutions[15] obtained for density, velocity, pressure, and temperature are similar to the solutions for a one-component gas. At high Reynolds numbers Re_* the flow remains nearly nonviscous at $r < r_+$. The parameters of the mixture at $r > r_+$ correlate well with the corresponding parameters in a one-component gas.[16] The major difference is in the shock wave front, where there is a considerable increase in the velocity of the light component. In the case of an argon-helium mixture ($\gamma = 5/3$, $n = 0.75$, $\varepsilon = 0.1$, $Sc = 0.333$, $Pr = 0.431$, $\beta = 0.377$, concentration of heavy component in flooded space $f_a = 0.5$), the maximum velocity of the helium species exceeds the limiting velocity of the mixture by a factor of 3.

As in the one-dimensional case,[17] concentration of the light component occurs in the leading front of the spherical shock wave.[15,18] The helium concentration in this area depends on the initial concentration of species and is practically constant with variation in the similarity parameter[8,9] $K_2 = Re_*(p_a/p_{s^\circ})^{1/2}$. The maximum values of concentration of the light component are $f \approx 0.5f_a$ at $f_a = 0.02$ and $K_2 = 1.24$. At $r > r_+$ and $K_2 \to 0$, the enrichment of the mixture with the heavy component inside the shock wave front was found.[15]

The flow structure in the inner supersonic region ($r < r_+$) could be studied in terms of the asymptotic analysis described above. Here, the flow remains close to ideal. The differences become significant in the regions with the large magnitudes of function derivatives near the critical source surface and the leading front of the shock wave.

Transonic Zone

The asymptotic solution in the transonic zone was found by Gusev and Riabov[15] in the case of binary mixtures. The method of deformable coordinates[10] was used.

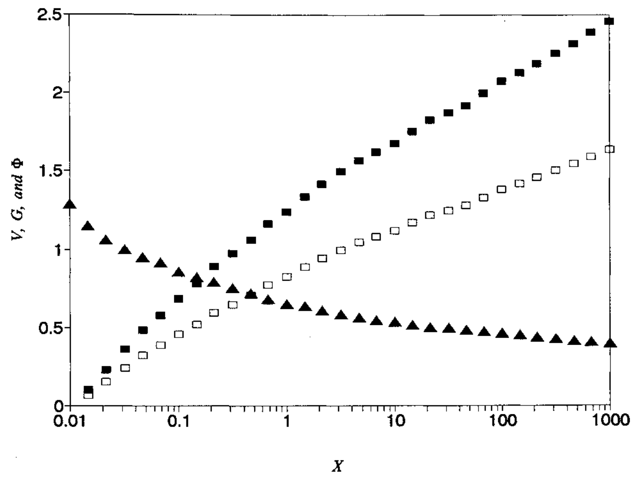


Fig. 2 Parameters V, G, and Φ (markers ■, □, and ▲, correspondingly) in transonic region of spherical flow of argon-helium mixture.

The flow parameters were made dimensionless with respect to their values on the critical surface of the ideal source. These new dependent and independent variables were introduced:

$$V = R^{1/3}(v - 1), \qquad G = R^{1/3}(1 - t)$$
$$\Phi = R^{2/3}(f - f_{*i}), \qquad X = R^{2/3}(1 - x) \tag{6}$$

Taking $R = 0.75Re_{*i} \to \infty$, we obtain the solution of the Navier-Stokes equations[15]:

$$V = 2\sqrt{\frac{X}{\gamma + 1}}\frac{I_{-2/3}(\eta) - I_{2/3}(\eta)}{I_{-1/3}(\eta) - I_{1/3}(\eta)} \quad \text{at} \quad X > 0$$
$$V = 2\sqrt{\frac{|X|}{\gamma + 1}}\frac{J_{-2/3}(\eta) - J_{2/3}(\eta)}{J_{-1/3}(\eta) + J_{1/3}(\eta)} \quad \text{at} \quad X < 0$$
$$G = (\gamma - 1)V, \qquad \eta = \frac{2\sqrt{\gamma + 1}}{3A}|X|$$
$$F_{*i} = \varepsilon + (1 - \varepsilon)f_{*i} \tag{7}$$
$$\Phi = \frac{3f_{*i}(1 - f_{*i})}{4Sc_{*i}}\left[\frac{1 - \varepsilon}{F_{*i}}\gamma - \beta_{*i}(\gamma - 1)\right]\frac{dV}{dX}$$
$$A = 1 + \frac{3(\gamma - 1)}{4\sigma_{*i}} + \frac{3f_{*i}(1 - f_{*i})}{4Sc_{*i}}$$
$$\times\left(\frac{1 - \varepsilon}{F_{*i}} - \frac{\gamma - 1}{2}\beta_{*i}\right)\left[\frac{1 - \varepsilon}{F_{*i}}\gamma - \beta_{*i}(\gamma - 1)\right]$$

The asymptotic solution (7) for argon-helium mixture ($f_{*i} = 0.5$) is shown in Fig. 2. The comparison of the solution given by Eq. (7) with a numerical solution was analyzed by Gusev and Riabov.[15]

Hypersonic Zone

In the analysis of the structure of a hypersonic flow region it is found that in the case of an ideal gas the expansions (1) are valid for the hypersonic flow parameters. The asymptotic solution of the Navier-Stokes equations in the hypersonic zone was found by introducing new variables[15]:

$$\Theta = tR^\lambda, \qquad W = (v - u_0)R^\lambda, \qquad \Psi = (f - f_{*i})R^\lambda$$
$$Z = xR^\omega, \qquad \lambda = 2\omega(\gamma - 1) \tag{8}$$

After substitution into the Navier-Stokes equations, and taking $R \to \infty$, we obtain[15]

$$\Theta = \left[\frac{u_0^2\gamma(\gamma - 1)(1 - n)}{[2\gamma - 1 - 2n(\gamma - 1)]Z} + \theta_1^{1-n}Z^{2(\gamma-1)(1-n)}\right]^{1/(1-n)}$$

$$W = -\frac{\Theta}{u_0(\gamma - 1)} - \frac{u_0\Theta^n}{Z} \tag{9}$$

$$\Psi = \frac{3\varepsilon f_{*i}(1 - f_{*i})}{4Sc_{*i}F_{*i}}\left(\frac{1 - \varepsilon}{F_{*i}} - \beta_{*i}\right)\Theta^{n-1}\frac{d\Theta}{dZ}$$

The expressions for Θ and W in Eq. (9) are the same as the solutions for a one-component gas.[10] As $Z \to \infty$, the solution (9) asymptotically approaches solution (1) for an ideal gas. The function Θ reaches its minimum value Θ_+ at a finite value $Z_+ > 0$, corresponding to the region of the leading front of a spherical shock wave. The function W varies in a similar way. These properties for argon-helium and nitrogen-hydrogen mixtures at $f_{*i} = 0.1$ are illustrated by Fig. 3.

It is noted that functions Θ and W do not contain a concentration of the components as a parameter. Following Eq. (9), the function Ψ is determined by the derivative $d\Theta/dZ$, and it should change sign at $\Theta \approx \Theta_+$. This phenomenon is illustrated by Fig. 4 for Ar-He (squares) and N_2-H_2 (triangles) mixtures. At $Z < Z_+$, the function Ψ is negative, and the enrichment of the mixture by the light component is observed in the front zone of a spherical shock wave. This effect can be useful for determination of the shock-front location in an experiment. The concentration of the light component has maximum value in this region under the condition $f_{*i} \approx 0.1$ for considered mixtures.

Using the numerical technique of Gusev and Riabov,[15] the spherical expansion of a binary gas mixture into a flooded space has been analyzed with the presence of a diffusion flux at infinite points. The numerical results were calculated for the cases of the expansion of helium into a space filled by argon, and the expansion of argon into a space filled by helium. The computational data indicates that in both cases the gas of the flooded space does not penetrate through the shock wave into the inner supersonic region of flow. This phenomenon was noted in experiments by Skovorodko and Chekmarev.[19]

The discussed phenomena and analytical expressions (6–9) were used for evaluation of the jet structure in aerodynamic applications.

The structure of the gas-mixture flow near the spherical nose probe was analyzed by Riabov[17] in terms of numerical solutions of the Navier-Stokes equations for nitrogen-hydrogen mixture. The increase in the temperature of an adiabatic wall, as well as nitrogen and hydrogen separation zones, were discussed.

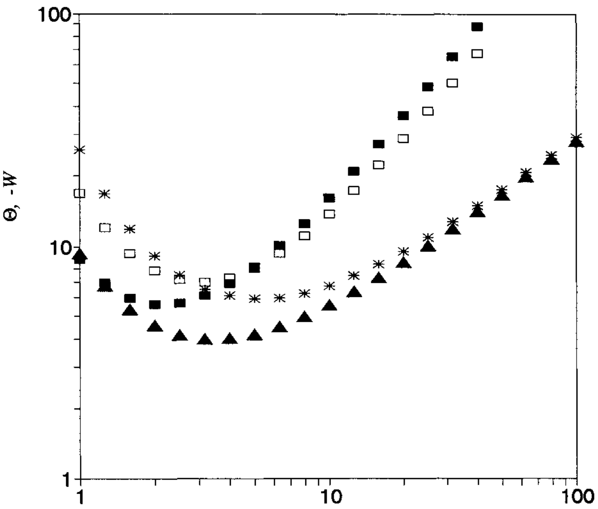


Fig. 3 Parameters Θ, W in hypersonic region of spherical flow of argon-helium (■, Θ, □, W) and nitrogen-hydrogen (▲, Θ, *, W) mixtures.

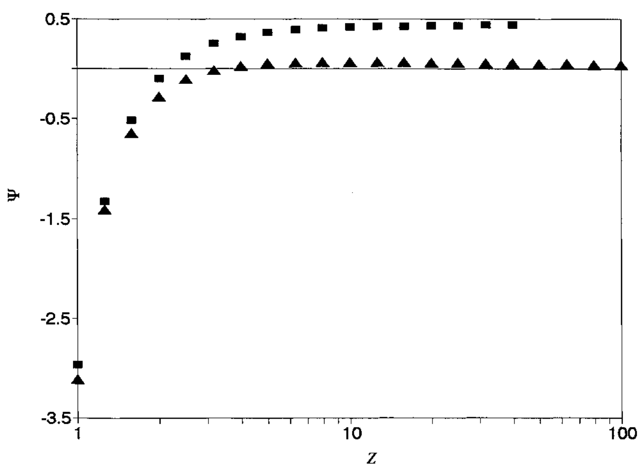


Fig. 4 Parameters Ψ in hypersonic region of spherical flow of argon-helium (■) and nitrogen-hydrogen (▲) mixtures.

Rotational Relaxation of a Freely Expanding Gas

Marrone[20] and Borzenko et al.[21] studied the translational-rotational (R-T) relaxation in the expansion of a molecular gas into a vacuum. A significant drop in the gas density downstream leads to a decrease in the number of molecular collisions and as a result, the departure of the rotational energy ε_R from the equilibrium value ε_R° is observed. Lebed and Riabov[22] studied another cause for the rotational energy departure, which could be explained in terms of quantum concepts. Because of the significant decrease in kinetic temperature T_t, the Messy adiabatic parameter,[23] describing energy transfer between highly excited rotational levels unable to relax, becomes larger than unity. Adiabatic collision conditions are realized at a certain temperature, which is a characteristic for each rotational level. At these conditions, the rotational transfer probability begins to decrease significantly,[23,24] and the relaxation time τ_R increases.

Using the technique of Lebed and Riabov,[22] the rotational-translational relaxation times were calculated for nitrogen (see Fig. 5) at conditions of aerodynamic experiments in underexpanded jets.

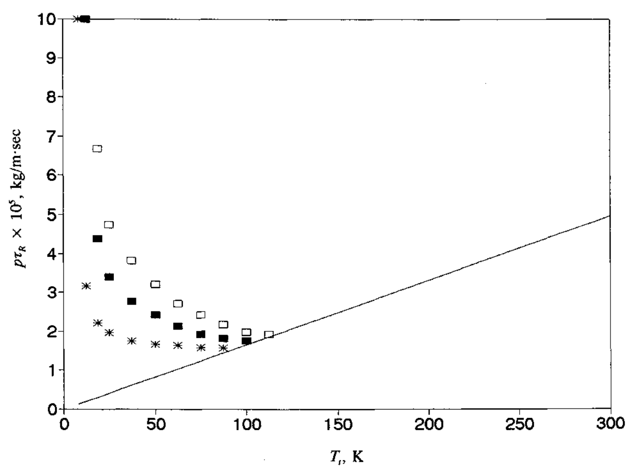


Fig. 5 Relaxation time τ_R of molecular nitrogen as a function of kinetic temperature T_t: solid line, Parker's model[25,26]; □, quantum rotational level[22] $j^* = 6$; ■, $j^* = 5$; *, $j^* = 4$.

The relaxation times calculated by Parker[25] are shown in Fig. 5 as solid line. This data could be approximated by the function[26]:

$$p\tau_R = CT_t^m \quad (10)$$

with the parameters $m = 1$ and $C = 1.65 \times 10^{-7}$ kg/m·s·K. The calculations based on the classical concept do not show a tendency of increasing $p\tau_R$ with the decrease of T_t under adiabatic rotational energy exchange conditions.

The average transition probabilities $\langle P_{j^*,j^*-2}\rangle$ per one gas-kinetic collision were calculated by the method of Lebed and Umanskii[24] for different rotational levels j^*. In the expansion of nitrogen, starting at a stagnation temperature $T_s = 300$ K, the maximum population of molecules occurs at levels j^* from 6 to 4. The result of calculating $p\tau_R$ from Eq. (2.1) of Ref. 22 for $j^* = 6$, 5, and 4 is shown in Fig. 5 (empty and filled squares, and asterisks, correspondingly). These values of $p\tau_R$ increase with decrease of T_t. At $T_t > 100$ K, the adiabatic condition breaks down, and the parameter $p\tau_R$ could be calculated by the Parker's model.[25]

For qualitative estimations, it is possible to replace the energy relaxation time by the relaxation time of the level j^*. This approximation method correctly represents the qualitative nature of the R-T nonequilibrium process, i.e., an increase of $p\tau_R$ with decreasing T_t.

Figure 6 shows the distributions of rotational temperature T_R along the axis of nitrogen jet. The result of using the classical mechanics concept[25,26] is shown there by the solid line. The curves marked by empty and filled squares, and asterisks were obtained in terms of the quantum concept[22] using the values of $p\tau_R$ for $j^* = 6$, 5, and 4, correspondingly. In the calculation it was assumed that the main R-T relaxation parameter was $K_* = \rho_* u_* r_*/(p\tau_R)_* = 2730$, which corresponds to $p_s r_j = 240$ Torr·mm and $T_s = 290$ K. In the case considered, the effect of viscosity and conductivity on the gas flow parameters was neglected.

The experimental results for T_R along the axis of a nitrogen jet were obtained by Marrone[20] (marker x) and Borzenko et al.[21] (triangles). The results were discussed in the study of Lebed and Riabov.[22] The experimental data are upper and lower bounds on the distribution of rotational energy along the flow. The computation results, based on the quantum concept of rotational energy exchange, correlate well with the experimental data.[21] The classical model[25,26] predicts the values of T_R, which do not correlate with experimental ones. The temperature distribution[22] in the isentropic expansion (i.e., $E_R = T_t$ and $\gamma = 1.4$) of nitrogen from a circular sonic nozzle is close to the E_R parameters predicted by the Parker's model.[25,26]

The experimental and computational results demonstrate the necessity of a consideration of quantum concepts in describing R-T relaxation in underexpanded jets.

Rotational Relaxation in Viscous Gas Flows

The combined effect of the rotational-translational relaxation and dissipative processes of viscosity and conductivity was analyzed by Riabov,[27,28] Lebed and Riabov,[29] Skovorodko,[30] Rebrov and Chekmarev,[31] and Molodtsov and Riabov.[32] The full system of the Navier-Stokes equations and the relaxation equation, based on the τ-approximation technique,[29] has been solved by the implicit technique utilizing the stabilization principle described by Riabov[28] in detail.

Solutions of the boundary value problem depend on the following parameters: Reynolds number Re_*, Prandtl number Pr, and p_a, T_{sa}, n, m, and K_*. The influence of these parameters on the nonequilibrium viscous diatomic gas flow from the sonic spherical source was studied with the help of the numerical computations. The results of these computations confirmed the earlier discovered delay[30,31] of the rotational temperature compared to the translational one. The rate of decrease of rotational temperature slows down with the gas expanding in the inner supersonic area of the flow that leads to its "frozen" value. Rotational-translational equilibrium never exists in front of the shock wave in such flow, and $T_R > T_t$.

As the result of gas compression in the shock wave, rapid increase of rotational temperature occurs. Following the point $T_R = T_t$ the corresponding values of translational temperature begin to increase too. As the gas expands in the subsonic area of the flow behind the shock wave, gradually the temperature reaches the value of T_a.

The distributions of T_R and T_t are shown in Fig. 7. The same figure displays the influence of different changes in pressure ratio p_*/p_a under the conditions $Re_* = 161.83$, $K_* = 28.4$, $Pr = n = 0.75$, and $T_{sa} = 1.2T_*$. Inviscid flow parameters T_R (solid lines) and T_t (dashed lines) were obtained by the method of Lebed and Riabov.[22]

As well as in the case of equilibrium flow,[11,15,16] there is an area of the starting point of expansion in which the distribution of parameters does not change when p_*/p_a increases, but due to dispersion of the shock wave the zone of nonequilibrium flow increases. An essential influence of viscosity and thermal conductivity is also noted in the differences of the translational temperature values from the values calculated by means of Euler equations. At the same time viscosity and thermal conductivity have a minor influence on the distribution of rotational temperatures in the inner supersonic

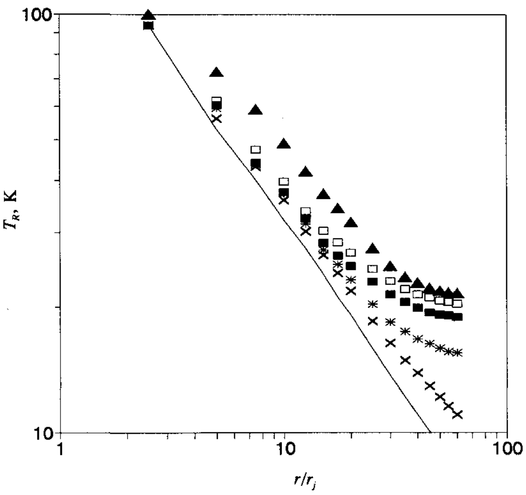


Fig. 6 Rotational temperature along the axis of nitrogen jet: solid line, Parker's model[25,26]; □, $j^* = 6$; ■, $j^* = 5$; *, $j^* = 4$; ×, experiment[20]; ▲, experiment.[21]

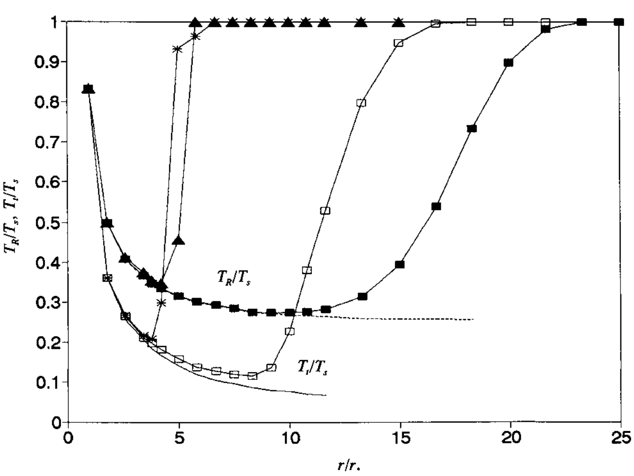


Fig. 7 Rotational T_R and translational T_t temperatures in spherical flow as a function of pressure ratio: ▲ and *, $p_*/p_a = 9.28$, ■ and □, $p_*/p_a = 92.8$.

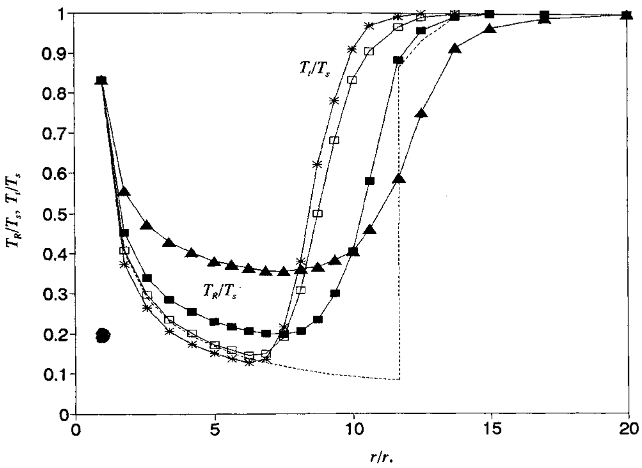


Fig. 8 Rotational T_R and translational T_t temperatures as functions of the main relaxation parameter K_*: ■ and □, K_* = 95.2; ▲ and *, K_* = 17.0.

region of the flow. This conclusion is supported by the results of the calculations[28] in which the influence of the value of the exponent n and Prandtl number Pr was studied.

The numerical analysis showed that the main parameters that influence the relaxation process are K_* and the approximation function $p\tau_R(T_t)$. This completely agrees with the results obtained by Lebed and Riabov,[22] and Skovorodko.[30] The decrease of the main parameter of relaxation K_*, when all other parameters are fixed values, leads to a faster frozen value of rotational temperature (see Fig. 8). The differences of the translational temperature from the corresponding equilibrium values (dashed line in Fig. 8) at $\gamma = 1.4$ are minor in supersonic zone. The results were considered at $Re_* = 161.83$, $p_*/p_a = 41.67$, $n = Pr = 0.75$. The increase of Re_* leads to a decrease of the relaxation zone near and in front of the shock wave. In the inner supersonic zone of the flow the value of both translational and rotational temperatures are close to the corresponding values calculated by means of Euler equations.[22]

The R-T nonequilibrium flowfield near the spherical-nose probe has been analyzed by Riabov[27] and Molodtsov and Riabov[32] under the conditions of the aerodynamic experiment in underexpanded jets.

The effect of frozen rotational energy can be significant for the prediction of heat transfer to the surface of the model,[27] for estimation of the perturbation zone near the model,[32] as well as for correct displacement of the model in the supersonic region of the jet.[28]

Aerodynamic Characteristics of Simple Shape Bodies

The similarity conditions for hypersonic flows in the transitional regime, which is between continuum media and free molecular flow, were defined by Gusev et al.[5] These studies offered the method of strongly underexpanded jets for aerodynamic experiments in order to receive experimental data in the broad range of changes of the main criterion of similarity, i.e., Re_0, in which the viscosity coefficient was calculated by means of stagnation temperature T_s. Due to this method, fundamental laws of hypersonic streamlining of bodies were discovered and a great experimental material about aerodynamic and thermal characteristics of different bodies was received by Gusev et al.[5,6,33,34]

Studying the dependency of aerodynamic characteristics of other similarity parameters is also of great interest. The parameters are temperature factor t_w; specific heat ratio γ, and viscosity coefficient approximation as a function of temperature, i.e., exponential approximation $\mu \sim T^n$. However, it is not always possible to conduct a direct comparison of experimental and computational data based on this set of parameters. The latter referred to the parameter γ, as the solution of the Boltzmann equation as built only for simple molecular models.

The results obtained permit the evaluation of the influence of the above-mentioned similarity criteria on the aerodynamic characteristics of the bodies of simple shapes in the transitional flow regime. The testing was performed in underexpanded viscous jets in a vacuum wind tunnel.

The presence of a nonuniform field in the expanding flow is considered here in terms of the experimental technique of Gusev et al.[6] and Nikolaev.[35] Relative difference of aerodynamic characteristics in the nonuniform flow from corresponding characteristics in the uniform flow can be evaluated by the parameter l/r, where l is the length of the model and is connected with the presence of axial gradient of density and the difference of the speed vector from the axial one. The method of the recalculation of aerodynamic characteristics of wide range of bodies applicable to vacuum wind tunnels was developed by Nikolaev,[35] and it leads to defining some cross section of the flow in which speed and similarity parameters required for the testing should be selected. In this study the parameters of upstream flow at the point of the flow corresponding to the middle of the model were used for the determination of the testing values of aerodynamic coefficients of simple shape bodies.

Demonstrated below are the results obtained in the strongly underexpanded flows having considered the above-mentioned advantages of their applications in an aerodynamic experiment. The testing was conducted in the vacuum wind tunnel with different gases: helium, argon, air, nitrogen, and carbonic acid at the stagnation temperature $T_s = 295$ K. The axisymmetric sonic nozzles ($M_j = 1$) with different radii of the critical cross sections r_j were used for obtaining the hypersonic flow. Plates and wedges were selected as working models. The coordinate of the front side of the Mach disk r_d was determined by the method described above having considered the recommendations of the study by Gusev.[12] The influence of viscous and nonequilibrium effects on the density and the flow velocity for all the gases was insignificant. Therefore, the main similarity criterion Re_0 and the dynamic pressure q were calculated using the nonviscous flow parameters obtained by the method of the characteristics in all the following processing of the testing data.

Results of Testing

Influence of Mach Number

The flow near the body of the fixed shape strives for the certain limited condition at the continuous increase of M. This condition can be reached for some bodies at comparatively moderate values of M_∞. Numerous exact solutions and testing data[6] on streamlining of such bodies, which were obtained at large values of Reynolds number, confirm this conclusion. The hypersonic stabilization regime remains the same for the transitional flow conditions.[5,6,33,34,36] The study of the influence of M_∞ on the aerodynamic characteristics of the bodies of simple shape is conducted at small values of Reynolds number and the constant values of other similarity parameters: Re_0, t_w, γ, and n.

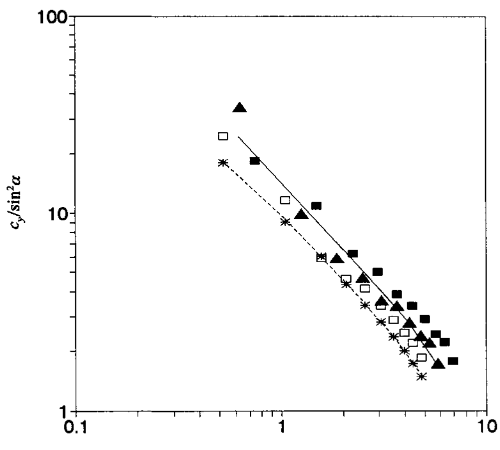
The regime of hypersonic stabilization[37] will even occur at $K = M_\infty\theta >> 1$ in the case of streamlining of the thin bodies when the angle θ between the generatrix of the body surface and the direction of the upstream flow becomes small enough. This regime will be realized at smaller values of M_∞, if the angle θ increases.

The results of such studies were obtained for the plate having relative thickness $\delta = h/l = 0.06$ at α. The testing was conducted in helium ($\gamma = 5/3$) at $Re_0 = 2.46$, and $t_w = 1$ (the wall temperature was $T_w = 295$ K in all experiments). The selection of helium as the working gas is explained by the constant value of exponent $n = 0.64$ within the range of temperature changes.[38] The dependency of the main aerodynamic coefficients on α was analyzed by Gusev et al.[6] It was found that the maximum difference from its asymptotic values, which correspond to the continuous increase $M_\infty \to \infty$, is observed for the drag coefficient of the plate at small angles of attack, when the hypersonic similarity parameter $K = M_\infty \sin \alpha$ becomes not large enough for the thin bodies.

The experimental data for aerodynamic coefficients c_y and c_x for the blunt plate $\delta = 0.06$ at α are shown in Figs. 9a and 9b, correspondingly. The results for the values of the Mach number $M_\infty = 7.5$ (empty squares), 9 (triangles), and 10.7 (filled squares) are presented in terms of the functions[37] $f_y = c_y/\sin^2\alpha$ and $f_x = (c_x - c_{x0})/\sin^3\alpha$, and the correlation parameter $K = M_\infty \sin \alpha$.

The length of the model was chosen as specific linear measure, and the planform area were selected for the calculation of the aerodynamic parameters c_x, c_y, and Re_0. The comparison of the testing data with the calculated ones, obtained by the direct simulation Monte Carlo (DSMC) technique[6] at $M_\infty = 9$, $Re_0 = 2.46$, $t_w = 1$, $\gamma = 5/3$, $n = 0.67$, is shown in the same figure.

The correlation of the results in terms of the functions[37] f_y and f_x is well acceptable. At different magnitudes of M_∞, the values of the functions are approximately the same at $K \geq 3$ or at $\alpha \geq 24$ deg. This is also proved by the calculated data of the free molecular flow (dashed lines for $M_\infty = 7.5$ and asterisks for $M_\infty = 10.7$) shown in the same figure. For the lift coefficient, the free-molecular flow data is independent of the Mach number, and the value $c_{f,FM}$ is less than the value c_f for transitional regime at $\alpha > 16$ deg. This effect was discussed by Gusev et al.[5,6]



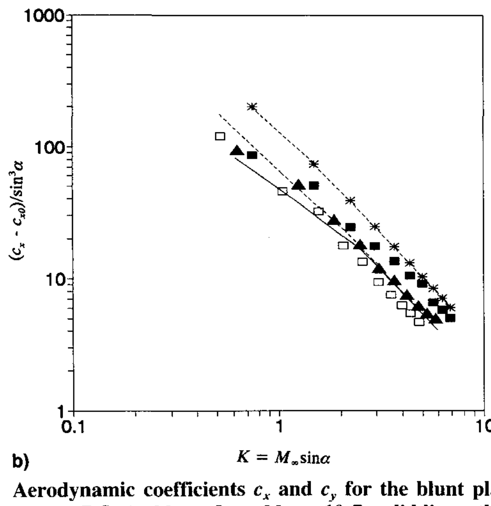


Fig. 9 Aerodynamic coefficients c_x and c_y for the blunt plate ($\delta = 0.06$): □, $M_\infty = 7.5$; ▲, $M_\infty = 9$, ■, $M_\infty = 10.7$; solid lines, the DSMC technique,[6] $M_\infty = 9$; dashed line, free-molecular regime,[39] $M_\infty = 7.5$; *, $M_\infty = 10.7$.

This study indicates that the principle of independence of the hypersonic flow of the Mach number will be true at the smaller values of M_∞ for the blunt bodies, not for the thin bodies at small angles of attack.

The dependency of the lift-drag ratio Y/X of the wedge ($2\theta = 40$ deg) on the angle of attack was obtained by testing in the helium flow at $Re_0 = 4$, $t_w = 1$, $M_\infty = 9.9$ (empty squares), and $M_\infty = 11.8$ (filled squares) (see Fig. 10). The base area of the wedge and its length were taken as the surface and linear measures. The obtained results indicate the weak dependency of aerodynamic coefficients on M_∞ in this regime. This is also proved by the calculated data of the free molecular flow (dashed line) shown in the same figure. The lift-drag ratio in the transitional regime is bigger than the ratio for the free molecular flow by a factor of 1.8.

The conducted research confirms the principle of hypersonic Mach number independence in the hypersonic transitional regime.[6]

Influence of the Specific Heat Ratio γ

While considering the issue of the influence of γ on the specific features of the hypersonic streamlining of the body in the transitional regime, it is necessary that the rest of the similarity parameters remain constant as the parameter γ changes. Thus, the pairs of such gases as helium-nitrogen, or nitrogen-carbonic acid are omitted from consideration at testing conditions. The dependencies of viscosity coefficient on temperature are different for the gases.[38] The nitrogen-argon pair is acceptable for testing.

The drag coefficient of the thin bodies will be proportionate to $(\gamma + 1)$ at the regime of hypersonic stabilization.[6] The identical conclusion is derived from the testing studies conducted with argon ($\gamma = 5/3$, $n \approx 1$) and nitrogen ($\gamma = 1.4$, $n \approx 1$). The dependencies of c_x of the wedge ($2\theta = 40$ deg) for Ar (filled squares) and N_2 (empty squares) are shown in Fig. 11 for a wide range of Re_0. At the same parameters of the upstream flow, testing data are compared with calculated data (solid line: $\gamma = 5/3$, $n = 1$; dashed line: $\gamma = 4/3$, $n = 1$) obtained by the DSMC technique[6] for two models of molecules (classical Maxwell and rough Maxwell spheres).

The influence of specific heat ratio on the drag coefficient is more significant for small values of $Re_0 < 10$. In the free molecular regime[6,39] ($Re_0 \to 0$) a small increase of c_x is observed as γ grows. This increase is caused by the impulse of the reflected molecules at $t_w = 1$. The degree of this influence was evaluated by Gusev et al.[6] as 3%. Identical dependency was found in the testing for transitional regime at $Re_0 < 10$ (about 5%). As the number Re_0 increases, this influence decreases.

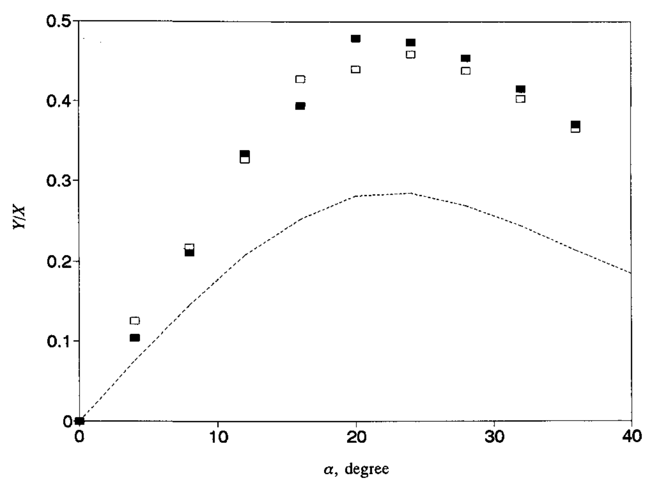


Fig. 10 Lift-drag ratio Y/X of the wedge ($2\theta = 40$ deg) at $Re_0 = 4$ and $M_\infty = 9.9$ (□) and 11.8 (■). Dashed lines, free-molecular flow characteristics.

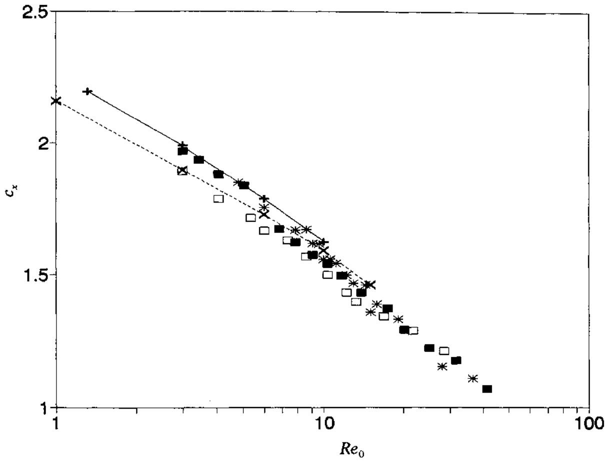


Fig. 11 Drag coefficient c_x for the wedge (2θ = 40 deg) for different gases (■, Ar; □, N_2; *, CO_2) vs the Reynolds number Re_0. Solid (n = 1, γ = 5/3) and dashed (n = 1, γ = 4/3) lines correspond to the DSMC technique data.[6]

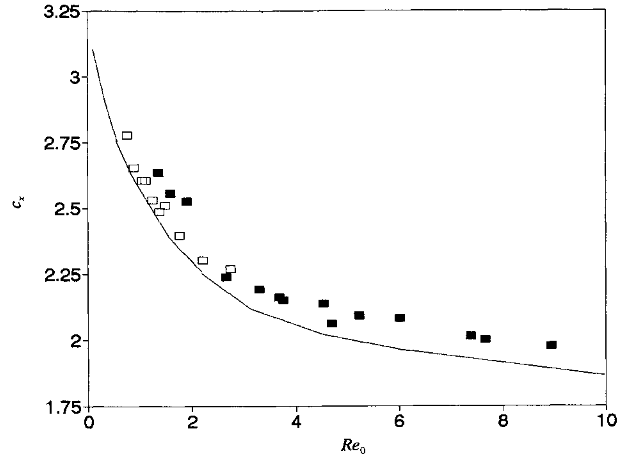


Fig. 13 Drag coefficient of the plate c_x at α = 90 deg as a function of Reynolds number Re_0. Experimental data: □, helium; ■, argon. The results of the DSMC technique[6]: n = 1, t_w = 1, 5 < M_∞ < 20, γ = 5/3.

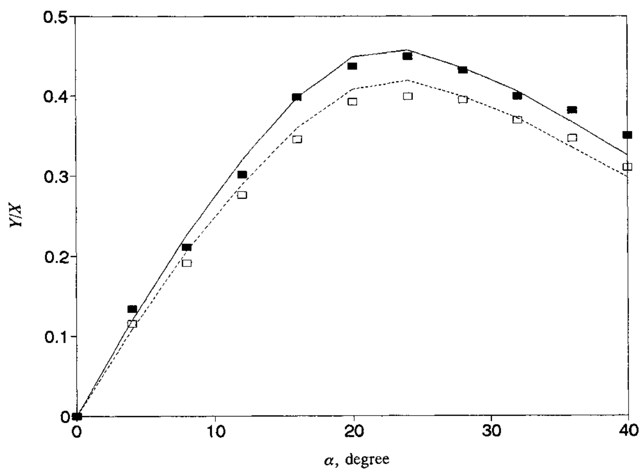


Fig. 12 Lift-drag ratio for the wedge (2θ = 40 deg) at Reynolds number Re_0 = 3. Experimental data: Ar (■) and air (□). The results of the DSMC technique[6]: dashed lines, for γ = 4/3, n = 1, M_∞ = 11; solid lines, for γ = 5/3, n = 0.83, M_∞ = 14.8.

For better correlation of experimental and computational[6] conditions at $\gamma < 1.4$, the testing was conducted for carbonic acid (γ = 9/7, asterisks in Fig. 11). The experimental data correlate well with the numerical results (dashed line) and the data for nitrogen and argon.

As mentioned above, the influence of the specific heat ratio on the aerodynamic characteristics of the bodies in free molecular flow depends on the normal component of the impulse of the reflected molecules, which is a function of γ. The same phenomenon was observed at transitional conditions[6] in the case of the plate at the angle of attack.

This phenomenon takes place in the case of streamlining of the wedge (2θ = 40 deg) at 0 deg < α < 40 deg and Re_0 = 3. The experimental data (filled squares for argon and empty squares for air) and calculated results by the DSMC technique[6] (solid lines for γ = 5/3, n = 0.83, M_∞ = 14.8, and dashed lines for γ = 4/3, n = 1, M_∞ = 11) are shown in Fig. 12. The correlation of the data demonstrates a significant difference (about 10%) in the values of Y/X.

Influence of the Viscosity Parameter n

The helium-argon pair is acceptable for testing to analyze the influence of viscosity parameter n, which is used in the approximation of viscosity coefficient $\mu \sim T^n$. It is noticed that the exponent n is closely related to the exponent s in the exponential law[14] of molecular interactions, and $n = \frac{1}{2} + 2/(s - 1)$. For Maxwell molecules, we have s = 5 and n = 1.

The exponents n for these monatomic gases (argon and helium) have approximately constant values and differ extremely ($n_{Ar} > n_{He}$) at T < 300 K.[38]

The testing of the plate at α = 90 deg (see Fig. 13) was conducted in these gases (□, helium; ■, argon) at t_w = 1, T_s = 295 K. The experimental data indicates that the insignificant increase (about 5%) of c_x occurs with the increase of the exponent n at $Re_0 \leq 2$.

The same study, based on the calculated results by the DSMC technique, was done theoretically by Gusev et al.[6] The experimental data correlates well with the computational results by the DSMC technique[6] at n = 1, t_w = 1, 5 < M_∞ < 20, γ = 5/3 (solid line).

The results of c_x of the plate at α = 90 deg indicated that the differences in functions $c_x(Re_0)$ for different molecular potentials s (or exponents n) are less than 3%. The experimental and computational results for disks, wedges, and plates were received by Gusev et al.[6] They indicate the insignificant influence (about 5%) of viscosity parameters on aerodynamic characteristics of the bodies in the hypersonic stabilization regime.

The influence of another similarity parameter, i.e., the temperature factor t_w, was analyzed by Gusev et al.,[6,33] and Tolstykh[40] in detail. The application of these studies for a prediction of hypersonic vehicle flight parameters in the Mars atmosphere conditions was developed by Riabov.[41]

Concluding Remarks

The methods used in this study allow the user to study effectively the hypersonic viscous flows of the monatomic and polyatomic gases in the underexpanded jets, and also the streamlining of bodies in the transitional regime between free-molecular and continuum regimes. Good correlation is noticed between testing data and calculated results obtained by the direct simulation Monte Carlo technique.[6]

Utilizing experimental and theoretical data, the principal correlations of the aerodynamic characteristics of the simple shape bodies (plate, wedge) were obtained in the transitional regime at different similarity parameters (Re_0, M_∞, γ, n).

In the hypersonic stabilization regime at $M_\infty \theta >> 1$, Re_0 is the main similarity criterium[5] for the correlation of the results, not only in the variations of M_∞, but also in the variations of the other similarity parameters (γ, n).

The influence of the parameters M_∞, γ, n significantly increases in the transitional regime at $M_\infty \sin\theta < 3$ and $Re_0 \leq 10$. The testing results of the streamlining of the thin plate at small angles of attack confirm this conclusion.

The acquired information about hypersonic viscous rarefied gas flows in underexpanded jets could be effectively used for investigation and prediction of the aerodynamic characteristics of hypersonic vehicle flights in complex atmospheric conditions of the Earth and Mars.

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