NUMERICAL INVESTIGATION OF THE FLOW OF NITROGEN PAST A SPHERE WITH ALLOWANCE FOR ROTATIONAL RELAXATION

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The supersonic flow of nitrogen past a sphere is studied in the framework of the complete Navier—Stokes equations and the relaxation equation with allowance for rotational—translational relaxation.

An influence of departure from rotational equilibrium on the flow structure near a sphere of radius R = 0.3 m can be expected for flight at altitudes 90-120 km [1]. In the experimental simulation of these flow regimes in wind tunnels, the nonequilibrium nature of the energy transfer must always be taken into account [2, 3]. The nonequilibrium excitation of the rotational degrees of freedom of the molecules can lead to an increase in the translational temperature and the thickness of the disturbed zone compared with the equilibrium case [4, 5].

We compare the results of numerical calculations with experimental data on the distribution of the nonequilibrium rotational energy of nitrogen in the neighborhood of the stagnation streamline and also on the distribution of the heat flux over the surface of the sphere.

1. The changes in the internal energy occur at finite rates, which can be characterized by relaxation times. For heavy molecules, the characteristic relaxation times of the rotational degrees of freedom are short [6, 7], so that at small Reynolds numbers and moderate velocities of the oncoming flow it can be assumed with good accuracy that the other forms of relaxation (for example, excitation of vibrations) are frozen. To calculate the time of establishment of equilibrium τ_R with respect to the rotational states one needs detailed information on the elementary process of transfer of rotational energy in a collision of molecules. At room temperature, about ten rotational levels of the nitrogen molecule are effectively populated, and to calculate τ_R in the framework of the kinetic equations for the populations one needs a huge amount of information on rotationally inelastic collisions, which at present is not available. In what follows, we have used a relaxation equation written down in the τ approximation [6]; we have calculated the rotational relaxation time τ_R by the method of [6, 7], and at very low temperatures with allowance for the departure from equilibrium in the oncoming flow and quantum effects [3].

The system of the complete nonstationary Navier—Stokes equations in an orthogonal curvilinear coordinate system s, n, φ (s is the coordinate measured along the profile of the body in the meridional plane, n is the normal to the surface of the body, and φ is the azimuthal angle) is derived in [8, 9] in divergence form for axisymmetric flow of a perfect gas. To describe the flow of nitrogen with allowance for rotational relaxation, the system must be augmented by a relaxation equation, the equation of state, and expressions for the total energy E of unit mass of the gas, the heat flux q, and the diffusion flux Ω of the rotational energy:

$$\frac{\partial}{\partial t}(\rho A\varepsilon_R)=\frac{1}{rH}\left[\frac{\partial}{\partial s}r(-\rho A\varepsilon_R u+\Omega_s)+\frac{\partial}{\partial n}rH(-\rho A\varepsilon_R v+\Omega_n)\right]+\frac{\rho\varepsilon_R^{\circ}-\rho\varepsilon_R}{p\tau_R(T_t)}ApK_R$$

$$p=\rho AT_t,\quad E=A(1.5T_t+\varepsilon_R)+0.5(u^2+v^2),\quad \mu=T_t^{\omega},\quad \Omega_s=\frac{A\mu}{\mathrm{Re}_0\,\mathrm{Sc}_R}\frac{1}{H}\frac{\partial\varepsilon_R}{\partial s};\quad \Omega_n=\frac{A\mu}{\mathrm{Re}_0\,\mathrm{Sc}_R}\frac{\partial\varepsilon_R}{\partial n}\tag{1.1}$$

$$q_s=\frac{A\mu\gamma}{\mathrm{Re}_0\,\mathrm{Pr}(\gamma-1)}\frac{1}{H}\frac{\partial T_t}{\partial s}+\Omega_s,\quad q_n=\frac{A\mu\gamma}{\mathrm{Re}_0\,\mathrm{Pr}(\gamma-1)}\frac{\partial T_t}{\partial n}+\Omega_n,\quad K_R=\frac{\rho_\infty u_\infty R}{p\tau_R(T_0)},\quad \mathrm{Re}_0=\frac{\rho_\infty u_\infty R}{\mu_0(T_0)},\quad A=\frac{CT_0}{u_\infty^2}$$

$$H=1+k(s)n,\quad r=r_w(s)+n\cos\theta$$

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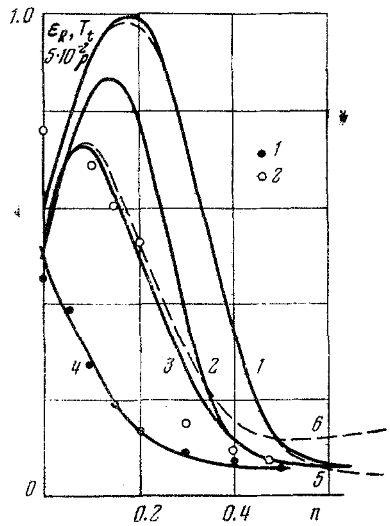

Fig. 1

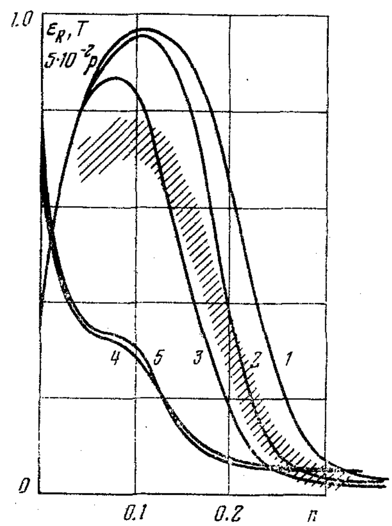

Fig. 2

Here, k(s) is the curvature, $r_w(s)$ is the distance from the symmetry axis to the considered point on the surface of the body, and θ is the angle between the direction of the velocity of the undisturbed flow and the tangent to the profile of the body in the meridional plane. All the variables in (1.1) are dimensionless; the linear dimensions are divided by the radius R of the sphere; the components u and v of the velocity vector along the directions of increase of the coordinates s and n, respectively, by the velocity of the undisturbed flow u_∞; the density ρ by ρ_∞; the pressure p by $\rho_\infty u_\infty^2$; the temperature T_t of the translational degrees of freedom, by the stagnation temperature T_0 in the undisturbed flow; the rotational energy ε_R of unit mass, by CT_0 (C is the gas constant); the viscosity μ, by $\mu_0(T_0)$; and the components of the vectors q and Ω, by $\rho_\infty u_\infty^3$.

Equations (1.1) were transformed to a form convenient for calculations and the region of integration chosen as in [8, 9].

On the outer boundary of the computational region, the gas flow was assumed to be undisturbed with parameters $\rho=1$, $T_t=T_{t\infty}$, $\varepsilon_R=\varepsilon_{R\infty}$, $u=\cos\theta$, $v=-\sin\theta$ for $0 \leqslant s \leqslant \pi/2$; for $\pi/2 < s \leqslant \pi$, we assumed fulfillment of the condition of "free flow" ($\partial/\partial n = 0$). On the central streamline, we used the symmetry condition of the flow; on the surface of the body (subscript w) we specified the condition of slip and the discontinuity of the temperature [10, 11]:

$$u=\left[\frac{2-a_1\alpha_1}{\alpha_1}\sqrt{\frac{\pi}{2}AT_t}\left(\frac{\partial u}{\partial n}-k(s)u\right)_w+0.84A\frac{\partial T_t}{\partial s}\bigg|_w\right]\frac{\mu}{p\,\mathrm{Re}_0}$$

$$\varepsilon_R=\varepsilon_{Rw}+\frac{2-a_3\alpha_3}{\alpha_3}\sqrt{\frac{\pi}{2}AT_t}\,\frac{\mu}{p\,\mathrm{Re}_0\,\mathrm{Sc}_R}\frac{\partial\varepsilon_R}{\partial n}\bigg|_w,\quad \varepsilon_{Rw}=T_w,\quad T_t=T_w+\frac{2-a_2\alpha_2}{\alpha_2}\frac{2\gamma}{\gamma+1}\sqrt{\frac{\pi}{2}AT_t}\,\frac{\mu}{p\,\mathrm{Re}_0\,\mathrm{Pr}}\frac{\partial T_t}{\partial n}\bigg|_w \tag{1.2}$$

In the calculations, we took $a_1 = 0.988$, $a_2 = 0.827$, $a_3 = 1$, $\gamma = 1.67$, $\mathrm{Sc}_R = 0.75$, Pr = 0.67, ω = 0.8. The gas molecules were assumed to have a diffuse reflection from the surface with coefficients of accommodation $\alpha_i = 1$, i = 1, 2, 3.

The numerical investigation was made by means of the conservative finite-difference scheme of [9], which is a modification of the difference scheme of [12]. A stationary solution to the problem was sought by the method of successive approximation by means of the stabilization principle.

2. We now analyze the flow in the viscous shock layer near the sphere. In Figs. 1 and 2, we give as examples the distribution of the translational (curves 1) and equilibrium (curves 2) temperatures, the nonequilibrium rotational energy (curves 3), and the density (curve 4) in the neighborhood of the stagnation streamline for two flow regimes: Re_0 = 15.3, 57.4, Mach numbers M_∞ = 9.18, 18.8, temperature factors $t_w = T_w/T_0$ = 0.3, 0.19, and T_0 = 293, 1600°K, respectively. The rotational degrees of freedom in the undisturbed flow were assumed to be in equilibrium. As was to be expected, at moderate

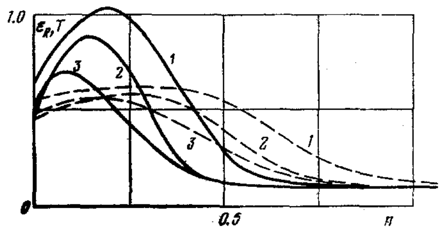

Fig. 3

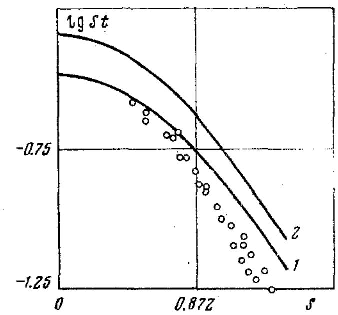

Fig. 4

Reynolds numbers, when the shock wave in front of the body is to a considerable extent smeared out, the rotational degrees of freedom can be strongly out of equilibrium, which leads to an increase in the translational temperature in the gas and the thickness of the shock layer compared with the equilibrium case. This conclusion agrees well with the results of [5]. Despite the strong difference between the nonequilibrium and equilibrium temperatures, the difference between the corresponding values of the density in the entire shock layer is slight (see curves 4 and 5 in Fig. 2). Note that the thickness of the disturbed region is determined by the basic relaxation parameter K_R. When it is increased, which can happen if the Reynolds number Re_0 is increased and the flow stagnation temperature T_0 is constant, the influence of the rotational disequilibrium will be less and the thickness of the disturbed region will decrease. Then the dissipative effects of viscosity, heat conduction, and diffusion will play a definite part.

The three-dimensional nature of the gas flow at the sphere has a strong influence on the distribution of the temperature characteristics of the flow. In Fig. 3, for two values of the coordinate s = 0.078 (continuous curves) and s = 1.18 (dashed curves), we have plotted the profiles of the translational (curves 1) and equilibrium (curves 2) temperatures, and also the nonequilibrium rotational energy (curves 3) oriented along the normal n to the surface of the body ($Re_0 = 14.4$, $M_\infty = 6.5$, $t_w = 0.34$, $T_0 = 1000^\circ K$). Despite the increase in the viscous shock layer, the relative thickness of the disturbed region changes only slightly. With increasing distance from the stagnation streamline, there is, because of the decreased temperature gradients, a less clearly expressed disequilibrium near the surface of the body, and this leads to a smaller difference of the heat flux from the equilibrium value.

For the considered flow regime, these features are also illustrated in Fig. 4, in which the Stanton number $St = q_w(\gamma_\infty - 1)/\rho_\infty u_\infty \gamma_\infty C(T_0 - T_w)$, calculated using the nonequilibrium heat flux (curve 1), is compared with the corresponding equilibrium values (curve 2) distributed over the surface of the sphere. It follows from the calculations that the nonequilibrium excitation of the rotational degrees of freedom decreases the specific heat flux to the wall compared with the equilibrium case; the corresponding differences in the distribution of the pressure and in the drag on the sphere do not exceed 5%.

In the simulation of these flow regimes in wind tunnels, especially in experiments using underexpanded jets, the gas flow which encounters the model may be strongly nonequilibrium [3], the relation $\varepsilon_{R\infty} > T_{t\infty}$ holding. The "separation" of the rotational temperature of the gas from the kinetic temperature is, on the one hand, a consequence of the pronounced reduction in the density of the gas downstream, which leads to a reduced number of collisions undergone by individual molecules, and, on the other, can be explained by the manifestation of quantum effects following the abrupt lowering of the kinetic temperature in the flow.

In Fig. 1, the dashed curve for one of the considered flow regimes gives the results of numerical calculation of nonequilibrium flow of nitrogen over a sphere ($\varepsilon_{R\infty}$ = 0.1305, $T_{t\infty}$ = 0.0345). At the leading edge of the disturbed region, as a result of compression of the gas, there is a rapid increase in the translational temperature (curve 5), and after the point $T_t = \varepsilon_R$ the corresponding values of the rotational energy begin to increase (curve 6). The flow then begins to recall the flow already considered, and near the surface the differences from the case of equilibrium flow become slight. Thus, the influence of the rotational—translational disequilibrium in the oncoming flow is concentrated in the front region of the viscous shock layer.

3. Let us now compare the results of the numerical investigation with the experimental data. The majority of the known experimental investigations into the hypersonic flow of a rarefied gas has been devoted to the heat fluxes and frictional stresses on a surface or the pressure distribution on a model. Despite the obvious value of these studies, it should be noted that they cannot give sufficient information on details of the structure of the complete flow field. With the development of electron-beam diagnostics, it has become possible to study in detail the local characteristics of the flow near investigated bodies. Thus, in one of the first studies in this direction [13], the local values of the density near the critical streamline of blunt bodies were determined. A detailed analysis based on comparison of experimental and numerical results on the density distribution was given in [5, 9, 14]. In the present paper, an analogous comparison, but with the results of [13] (points 1) is shown in Fig. 1 (curve 4). The agreement between the results is good, and a certain difference at the leading stagnation point of the sphere can be explained, on the one hand, by the strong influence of secondary electrons in the experimental investigations [2, 13] and, on the other, by a certain approximation in the specification of the boundary conditions for the problem.

It should be noted that the results of measurement of the density are evidently less suitable for investigating nonequilibrium flow over a blunt body. We shall therefore make a comparison of the results of the numerical calculations with the experimental data of [2, 15], in which measurements were made of the profile of the nonequilibrium rotational energy of nitrogen near the stagnation streamline of a blunt body with spherical nose. The results of the comparison are shown in Fig. 1 (the points 2 are the experimental data of [15]) and also in Fig. 2 (the hatched region corresponds to the results of [2]). To an accuracy of 20%, the agreement can be regarded as satisfactory.

In Fig. 4, we compare the results of the numerical solution with the experimental data (points) on the distribution of the heat fluxes on a sphere obtained in a rarefied hypersonic gas flow [16]. The tests were made in a vacuum wind tunnel at $M_\infty = 6.6$, $Re_0 = 14.4$, $T_0 = 1000^\circ K$, $t_w = 0.315$. The local heat transfer on the sphere was determined by means of heat-indicating coverings. Note the good correlation between the experimental data and the calculated dependence.

The analysis of the shock-layer structure based on comparison of the experimental data for the rotational energy with the solutions to the Navier—Stokes equations simplified under the assumption of a locally self-similar nature of the flow in the neighborhood of the stagnation streamline for both the equilibrium [14] and nonequilibrium situation [5], and also with the solutions of the complete Navier—Stokes equations and the relaxation equation indicates the applicability of the latter equations.

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