The variation in static pressure along the shell (lower part of the figure) also agrees well with the experiment. Points 3 are the pressure measured along the wall of the shell and points 4 are the pressure found by measuring the transverse fields. The same total pressure ratio $p_{s0}^*/p_{p0}^* = 0.06727$ was taken in the calculation as in [7]. The value of $k = 0.232$ obtained by calculation agrees well with the experimentally determined value of $k = 0.24$.

The authors thank A. N. Kraiko for useful discussions and attention to the work.

LITERATURE CITED

1. V. M. Puzyrev and R. K. Tagirov, "Calculation of flow in ejector nozzles," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1974).
2. W. L. Chow and A. L. Addy, "Interaction between primary and secondary streams of supersonic ejector systems and their performance characteristics," AIAA J., 2, No. 4 (1964).
3. W. L. Chow and P. S. Yeh, "Characteristics of supersonic ejector systems with nonconstant area shroud," AIAA J., 3, No. 3 (1965).
4. J. M. Hardy, "Étude théorique d'une tuyère convergente—divergente biflux." Aeronaut. Astronaut., No. 37 (1972).
5. M. Ya. Ivanov, A. N. Kraiko, and N. V. Mikhailov, "The method of through calculation for two-dimensional and three-dimensional supersonic flows, I," Zh. Vychisl. Mat. Mat. Fiz., 12, No. 2 (1972).
6. N. L. Efremov and R. K. Tagirov, "Calculation of the base pressure in ejector nozzles of different lengths at a zero coefficient of ejection," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1974).
7. J. Paulon, "Étude théorique et experimental de la coexistence de deux flux dans un canal de section constance," AGARD, Current Paper No. 91 (1972).

SIMILARITY OF FLOWS IN STRONGLY UNDEREXPANDED JETS OF VISCOUS GAS

V. N. Gusev, T. V. Klimova, and V. V. Ryabov

UDC 533.6.011.8:532.525.2

The fulfillment of the conditions formulated in [1] for the similarity of flows in strongly underexpanded jets of a viscous, thermodynamically ideal gas is studied. The limits of applicability of these conditions are established on the basis of exact solutions of the one-dimensional Navier—Stokes equations and experimental investigations.

§1. Let us consider strongly underexpanded jets of a viscous, thermodynamically ideal gas escaping from arbitrary nozzles into a quiescent medium. The necessary conditions for the similarity of such flows were formulated in [1] on the basis of dimensionality theory and are expressed as equalities of the following similarity criteria:

$$M_j,\ K_2 = \mathrm{Re}_j\sqrt{p_\infty/p_{0j}},\ U_j\sqrt{\rho_\infty/p_\infty},\ \mu_j/\mu_\infty \tag{1.1}$$
$$M_j = U_j/\sqrt{\gamma_j R T_j},\ \mathrm{Re}_j = \rho_j U_j d_j/\mu_j,\ p_{0j} = p_j(1+0.5(\gamma_j-1)M_j^2)^{\gamma_j/(\gamma_j-1)}$$

Here p, ρ, T, U, and μ are the pressure, density, temperature, velocity, and coefficient of viscosity, respectively, the index j refers to the parameters in the initial cross section of the jet, the index ∞ refers to the parameters at infinity, and d_j is the characteristic size of the initial cross section of the jet.

The distributions of the stream parameters in the initial cross sections of the jets, normalized to the respective characteristic values, are assumed to be identical while the constants determining the physical properties of the gases in the jets and in the surrounding space, such as the ratio of specific heats γ, the exponent n in the law $\mu \sim T^n$, the Prandtl and Schmidt numbers, etc., are considered to be included among the similarity criteria.

Moscow. Translated from Izvestiya Akademii Nauk SSSR, Mekhanika Zhidkosti i Gaza, No. 6, pp. 117-125, November-December, 1978. Original article submitted November 14, 1977.

 0015-4628/78/1306-0886\$07.50

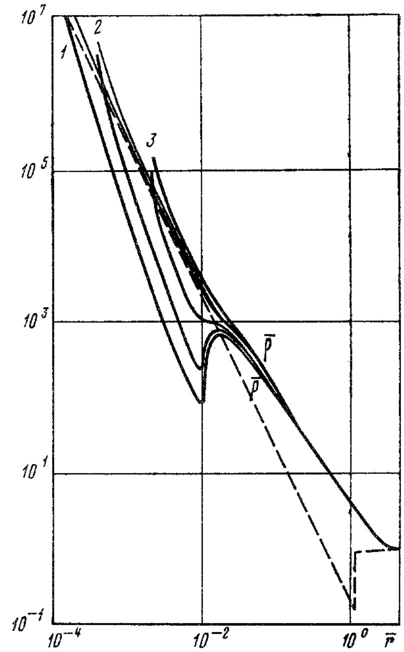

Fig. 1

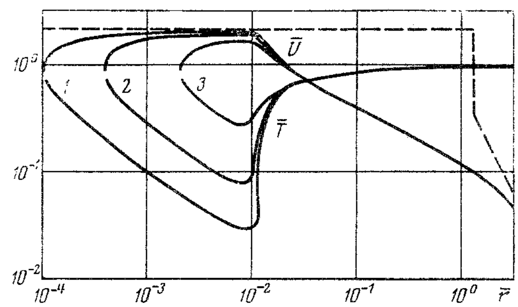

Fig. 2

If the physical properties of the gases in the jets and in the surrounding space are the same, the system of similarity criteria (1.1) is shortened to three:

$$M_j,\ K_2,\ T_{0j}/T_\infty,\ T_{0j}=T_j(1+0.5(\gamma-1)M_j^2) \tag{1.2}$$

The dimensionless dependent and independent variables of the problem are written in the form

$$\bar U=U/\sqrt{\gamma_\infty RT_\infty},\ \bar p=p/p_\infty,\ \bar\rho=\rho/\rho_\infty,\ \bar T=T/T_\infty$$
$$\bar x=\frac{x}{d_j}\sqrt{\frac{p_\infty}{p_{0j}}},\quad \bar y=\frac{y}{d_j}\sqrt{\frac{p_\infty}{p_{0j}}},\quad \bar z=\frac{z}{d_j}\sqrt{\frac{p_\infty}{p_{0j}}} \tag{1.3}$$

The similarity law formulated above is obtained in the limiting case when $p_\infty/p_{0j}\to 0$. But the appearance of regions in which the flow is entirely determined by the assignment of the initial data in the exit cross section of the nozzle and corresponds to the discharge of gas into a vacuum ($p_\infty=\rho_\infty=0$) is characteristic of strongly underexpanded jets in such a limiting transition. It is obvious that the use of the characteristic parameters of the surrounding medium as the scales in these regions can lead to an infinite increase or decrease in the dimensionless variables as $p_\infty/p_{0j}\to 0$. Physically this is connected with the fact that the flow inside the jet bounded by shock waves becomes infinitely overexpanded relative to the outside pressure p_∞.

For a rigorous approach in this region a transition to a new system of characteristic parameters of the problem, which can become the critical values of the stream parameters at M = 1 (denoted below by the index *), is necessary. In this case the system of similarity criteria and dimensionless dependent and independent variables is written in the form

$$M_j,\ \mathrm{Re}_d=\rho_*U_*d_j/\mu_*$$
$$U'=U/U_*,\ p'=p/p_*,\ \rho'=\rho/\rho_*,\ T'=T/T_* \tag{1.4}$$
$$x'=x/d_j,\ y'=y/d_j,\ z'=z/d_j$$

On the other hand, experimental investigations (see [10], for example) show that for some parameters similarity in the variables (1.3) continues to be satisfied inside the jet. In this connection it is interesting to establish the conditions under which a transition to the new variables in this region becomes necessary.

§2. First let us consider the fulfillment of the similarity conditions formulated above on the example of the spherical expansion of a gas into a flooded space (M = 1). The choice of this flow is explained, on the one hand, by the presence of an exact solution of the Navier–Stokes equations [2] and, on the other hand, by the equivalence of the flow in the hypersonic region of a jet to one-dimensional flow [3].

We separate the test region of the one-dimensional flow under consideration by the coordinate r_+ at which the stream parameters are extremal into a supersonic region $r < r_+$ and a region $r > r_+$ of transition of supersonic flow into subsonic flow. In the case of an ideal gas this flow will consist of supersonic and subsonic branches of an ideal source separated by a spherical shock wave whose position is determined by the coordinate r_+. As $p_\infty/p_{0*} \to 0$

$$\bar{r}_+ = \frac{r_+}{r_*}\sqrt{\frac{p_\infty}{p_{0*}}} = \left(\frac{2\gamma}{\gamma+1}\right)^{1/2}\left(\frac{\gamma-1}{\gamma+1}\right)^{(\gamma+1)/4(\gamma-1)}\left[\frac{(\gamma+1)^2}{2\gamma(\gamma-1)}\right]^{\gamma/2(\gamma-1)} \tag{2.1}$$

Here the following equations are valid for the stream parameters:

At $r < r_+$

$$\begin{aligned} &U'\left(\tfrac{1}{2}(\gamma+1)-\tfrac{1}{2}(\gamma-1)U'^2\right)^{1/(\gamma-1)} = r'^{-2},\\ &\rho' = \left(\tfrac{1}{2}(\gamma+1)-\tfrac{1}{2}(\gamma-1)U'^2\right)^{1/(\gamma-1)}\\ &T' = \left(\tfrac{1}{2}(\gamma+1)-\tfrac{1}{2}(\gamma-1)U'^2\right),\\ &p' = \left(\tfrac{1}{2}(\gamma+1)-\tfrac{1}{2}(\gamma-1)U'^2\right)^{\gamma/(\gamma-1)} \end{aligned} \tag{2.2}$$

at $r > r_+$

$$\begin{aligned} &\left(\tfrac{1}{2}(\gamma+1)\right)^{\frac{1}{2}(\gamma+1)/(\gamma-1)}\bar{U}\left(1-\tfrac{1}{2}(\gamma-1)\bar{U}^2\right)^{1/(\gamma-1)} = \bar{r}^{-2},\\ &\bar{\rho} = \left(1-\tfrac{1}{2}(\gamma-1)\bar{U}^2\right)^{1/(\gamma-1)}\\ &\bar{T} = \left(1-\tfrac{1}{2}(\gamma-1)\bar{U}^2\right),\ \bar{p} = \left(1-\tfrac{1}{2}(\gamma-1)\bar{U}^2\right)^{\gamma/(\gamma-1)} \end{aligned} \tag{2.3}$$

In accordance with the two chosen systems of characteristic parameters the former equations are written in the variables (1.4), in which $r' = r/r_*$, and the latter are written in the variables (1.3), in which $\bar{r} = rr_*^{-1}\sqrt{p_\infty/p_{0*}}$. When $p_{0*} \gg p_\infty$ the coordinate $r_+ \gg r_*$ and in the hypersonic flow region from Eqs. (2.2) we get the asymptotic expressions

$$\begin{aligned} &U' = \sqrt{(\gamma+1)/(\gamma-1)} + \ldots,\quad \rho' = \sqrt{(\gamma+1)/(\gamma-1)}\,r'^{-2} + \ldots\\ &T' = \left((\gamma-1)/(\gamma+1)\right)^{\frac{1}{2}(\gamma-1)} r'^{-2(\gamma-1)} + \ldots\\ &p' = \left((\gamma-1)/(\gamma+1)\right)^{\frac{1}{2}\gamma} r'^{-2\gamma} + \ldots \end{aligned} \tag{2.4}$$

Changing to the variables (1.3) in (2.4), we obtain

$$\begin{aligned} &\bar{U} = \sqrt{2/(\gamma-1)} + \ldots,\quad \bar{\rho} = \left(2/(\gamma+1)\right)^{1/(\gamma-1)}\sqrt{(\gamma-1)/(\gamma+1)}\,\bar{r}^{-2} + \ldots\\ &\bar{T} = \frac{2}{(\gamma+1)}\left((\gamma-1)/(\gamma+1)\right)^{\frac{1}{2}(\gamma-1)}(p_\infty/p_{0*})^{(\gamma-1)}\bar{r}^{-2(\gamma-1)} + \ldots\\ &\bar{p} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}\left((\gamma-1)/(\gamma+1)\right)^{\frac{1}{2}\gamma}(p_\infty/p_{0*})^{(\gamma-1)}\bar{r}^{-2\gamma} + \ldots \end{aligned} \tag{2.5}$$

It follows from (2.5) that as $p_\infty/p_{0*} \to 0$ the pressure and temperature in the hypersonic flow region are negligibly small in comparison with their values at infinity. Consequently, as already mentioned above, their representation in this region must be done in the variables (1.4). For the velocity and density such a division is not necessary in the hypersonic approximation [see Eqs. (2.3) and (2.5)]. The distribution of these parameters at $\gamma = 1.4$ in the variables (1.3) is given in Figs. 1 and 2 dashed lines ($K_2 = \infty$).

A similar flow pattern is also retained in the presence of viscosity. At $r < r_+$ the flow continues not to depend on the conditions at infinity, and at relatively high values of the Reynolds number it remains nearly ideal. This is confirmed by the results of numerical calculations [4], and also follows from an asymptotic solution, which is valid in the hypersonic region of flow [5].

The fulfillment of the similarity conditions at $r > r_+$ is illustrated by Figs. 1 and 2, in which the results of numerical calculations with $K_2 = 0.061$, $\gamma_* = \gamma_\infty = 1.4$, $T_{0*}/T_\infty = 1$, $n_* = n_\infty = 1$ are presented. In these figures curves 1-3 correspond to $Re_* = \rho_* U_* r_*/\mu_* = 565$, 157, and 30. Despite the fact that the data presented were obtained at finite values of the pressure drop p_∞/p_{0*}, the similarity in the variables (1.3) is satisfied rather well in the subsonic region of flow. The corresponding asymptotic solutions in this region as $p_\infty/p_{0*} \to 0$ are given in [5] for $T_{0*}/T_\infty = 1$ and in [4] for $T_{0*}/T_\infty \neq 1$.

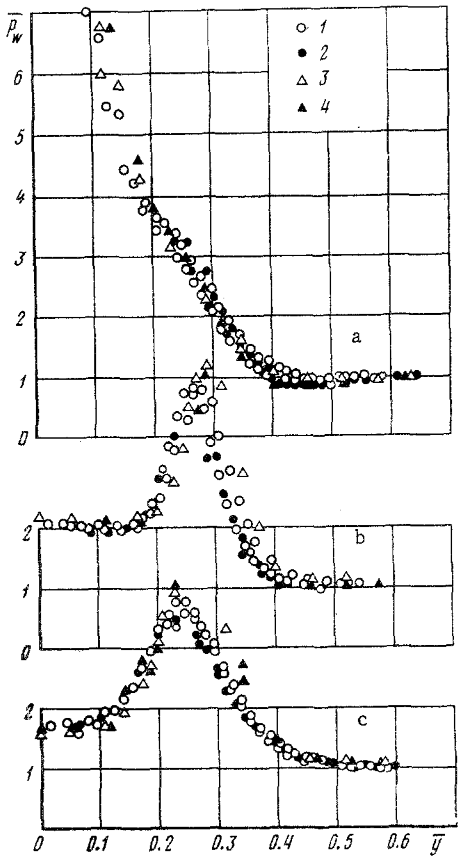

Fig. 3

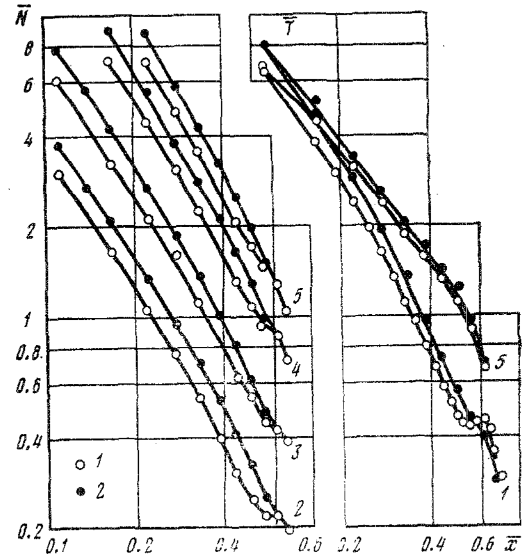

Fig. 4

In the supersonic region of flow the functions presented in Figs. 1 and 2 fan out. But the pressure and temperature in the hypersonic region of flow decrease, approaching zero, as $p_\infty/p_{0*} \to 0$, just as in the case of an ideal gas. A change to the criteria (1.4) is necessary for them. At the same time, such a division is not needed for the velocity and density in this region. With an error of Re_*^{λ}, $\lambda = 2(\gamma-1)\omega$, $\omega = [2\gamma-1-2n(\gamma-1)]^{-1}$, they can be represented in the variables (1.3) in the entire flow field except for the vicinity of the critical section of the source.

As the pressure drop p_∞/p_{0*} increases, the supersonic region of flow decreases, the velocity in it becomes less than its limiting value (see Fig. 2), and here a change to the criteria (1.4) is now necessary for all the stream parameters.

It should be noted that the mode of flow considered above, at a value of the similarity parameter K_2 as small as desired, refers to that level of rarefaction at which many of the gasdynamic criteria of a continuous medium in the stream vanish. This is indicated, for example, by the monotonic variation of the gas density. The existence of this mode is confirmed by the results of measuring the density in strongly underexpanded jets [6].

§3. As already mentioned, the investigation of the one-dimensional expansion of a viscous gas carried out above can be used in an analysis of the flow in the hypersonic region of a strongly underexpanded jet. The mutual equivalence of these flows in the isentropic case was demonstrated in [3]. Its rigorous proof in the presence of viscosity is presented below.

Let us consider the steady discharge of a strongly underexpanded jet from an axisymmetric nozzle. For a nonviscous gas the Euler equations admit the following asymptotic solution:

$$u' = u_0 + u_1\left[\frac{r_*'(\varphi)}{r'}\right]^{2(\gamma-1)} + \ldots,$$
$$v' = v_1\left[\frac{r_*'(\varphi)}{r'}\right]^{2(\gamma-1)} \frac{d\ln r_*'(\varphi)}{d\varphi} + \ldots, \tag{3.1}$$

$$
\rho' = \frac{1}{u_0}\left[\frac{r_*'(\varphi)}{r'}\right]^2 + \ldots, \quad T' = \theta_1\left[\frac{r_*'(\varphi)}{r'}\right]^{2(\gamma-1)} + \ldots,
$$

$$
p' = \frac{\theta_1}{u_0}\left[\frac{r_*'(\varphi)}{r'}\right]^{2\gamma} + \ldots, \tag{3.1}
$$

$$
u_0 = \sqrt{\frac{\gamma+1}{\gamma-1}}, \quad u_1 = -\frac{\theta_1}{\sqrt{\gamma^2-1}}, \quad v_1 = -\frac{2\theta_1}{[1-2(\gamma-1)]u_0},
$$

$$
\theta_1 = \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2(\gamma-1)}
$$

Here u and v are the radial and tangential velocity components, respectively, $r' = r/r_j$, r_j is the radius of the nozzle exit cross section, $r_*'(\varphi) = r_*(\varphi)/r_j$ is some arbitrary function of φ, and the dependent variables are made dimensionless relative to their critical values at M = 1.

From a comparison of Eqs. (2.4) and (3.1) and in accordance with the results of [3] it follows that at large distances from the nozzle exit cross section the isentropic flow in a jet asymptotically approaches, along each ray $\varphi = \text{const}$, the flow from some source having an intensity which varies from ray to ray. In order to obtain the dependence $r_*(\varphi)$ of the radius of the critical cross section of the equivalent source on the angle φ one must join the resulting solution with the solution for small r. The results of numerical calculations by the method of characteristics are usually used for this purpose.

At distances $r' = O(\mathrm{Re}^{\omega})$ from the source in the flow dissipative processes become important and the asymptotic expansion (3.1) loses force in this region. To find the solution here we use the Navier–Stokes equations in a spherical coordinate system having the origin in the nozzle exit cross section, and we change in them to the new dependent and independent variables

$$
W = \frac{-u_0+u'}{\alpha^{\lambda}}, \quad V = \frac{v'}{\alpha^{\lambda}}, \quad \Theta = \frac{T'}{\alpha^{\lambda}}, \quad X = \frac{r_*'(0)}{r'\alpha^{\omega}}
$$

$$
\left(\alpha = \frac{4}{3\,\mathrm{Re}\, r_*'(0)}, \quad \mathrm{Re} = \frac{\rho_* U_* r_j}{\mu_*}\right)
$$

After the change and the transition to $\alpha \to 0$, we obtain

$$
r^{\circ 2}\frac{\partial W}{\partial X} + nu_0\frac{\Theta^{(n-1)}}{X}\frac{\partial\Theta}{\partial X} - 2\frac{u_0\Theta^n}{X^2} = -\frac{r^{\circ 2}}{\gamma u_0}\left(\frac{\partial\Theta}{\partial X} + \frac{2}{X}\Theta\right)
$$

$$
r^{\circ 2}\frac{\partial\Theta}{\partial X} + u_0(\gamma-1)\left(r^{\circ 2}\frac{\partial W}{\partial X} + \frac{nu_0}{X}\Theta^{(n-1)}\frac{\partial\Theta}{\partial X} - \frac{u_0}{X^2}\Theta^n\right) = 0 \tag{3.2}
$$

$$
r^{\circ 2}\left(\frac{\partial V}{\partial X} - \frac{V}{X}\right) - \frac{1}{\gamma u_0 X}\frac{\partial}{\partial\varphi}(r^{\circ 2}\Theta) + \frac{nu_0}{2}\frac{\Theta^{n-1}}{X^2}\frac{\partial\Theta}{\partial\varphi} = 0
$$

Here $r^{\circ} = r_*(\varphi)/r_*(0)$.

Integration of the first two equations of system (3.2) with $n \neq 1$ gives

$$
W = -\frac{\Theta}{u_0(\gamma-1)} - \frac{u_0\Theta^n}{r^{\circ 2}X}
$$

$$
\Theta = \left[\frac{\gamma(\gamma+1)(1-n)\omega}{r^{\circ 2}X} + \theta_1^{1-n}(r^{\circ}X)^{2(\gamma-1)(1-n)}\right]^{1/(1-n)}
$$

When $\varphi = 0$ this solution coincides with the solution of [5]. Consequently, with the accuracy adopted here the viscous flow along the axis of an axisymmetric jet at large distances from the nozzle exit cross section asymptotically approaches the one-dimensional flow from a spherical source. Here the following equations from the one-dimensional solution will be valid for the distribution of the parameters:

$$
U' = u_0 + R^{-\lambda}W, \quad \rho' = R^{-2\omega}X^2/(u_0 + R^{-\lambda}W)
$$

$$
T' = R^{-\lambda}\Theta, \quad p' = R^{-2\omega-\lambda}\theta X^2/(u_0 + R^{-\lambda}W)
$$

$$
X = R^{\omega}r_*'(0)/r', \quad R = {}^3/_4\, r_*'(0)\,\mathrm{Re}
$$

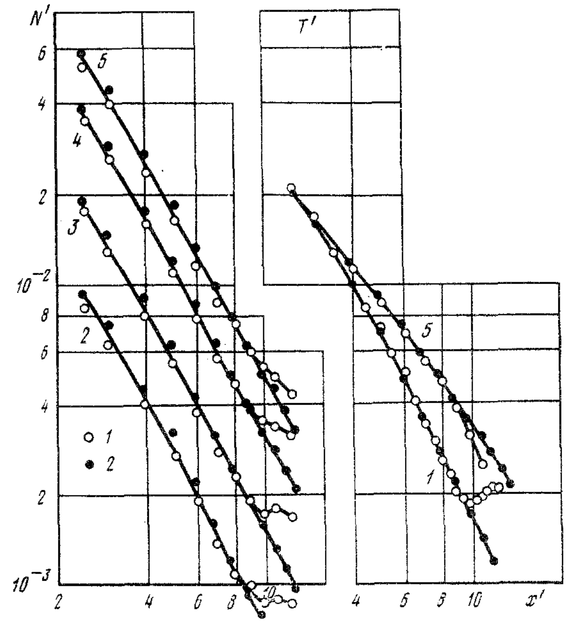

Fig. 5

In these equations for $r_*^{\prime}(0)$ there are numerous approximate functions containing M_j and γ. The equations are written in the variables (1.4). Just as in the case of an ideal gas, however, when $|W\bar{R}-\gamma| \ll u_0$ Eqs. (2.4) remain valid for the velocity and density, and a change to the variables (1.3) is possible for the variables with the adopted accuracy.

A corresponding analysis of the influence of viscosity on the gas flow near the jet axis, based on the one-dimensional analogy, was given in [7]. There also they noted the rather good agreement of some experimental data with the similarity law of [1], such as for the coordinate r_+ at which the density is extremal and for the density ρ_+ itself.

A more detailed experimental test of the similarity can be found in [8-10], in which the similarity criterion K_2 was also used in the treatment of the experimental data with the accuracy of the definitions. From these investigations it also follows that for moderately low values of the pressure drop p_∞/p_{0j} the similarity in the variables (1.3) is retained for the density ρ and the velocity head p_0' which is proportional to ρU^2 in the hypersonic region of a jet, in the entire flow field except for the vicinity of the exit cross section of a sonic nozzle.

§4. In conclusion, let us consider the interaction between strongly underexpanded jets and bodies. Here also one must distinguish two cases. When the outside pressure p_∞ has an appreciable influence on the disturbed flow near the body, the similarity conditions will be satisfied when there is equality, in addition to the criteria (1.1), of the parameters [1]

$$K_1 = L d_j^{-1} \sqrt{p_\infty / p_{0j}},$$
$$T_w / T_{0j}$$

where L is the characteristic size of the body over which the flow occurs and T_w is its temperature.

In this case the values of the Mach and Reynolds numbers will be the same at the similar points of the body over which the flow occurs ($\bar{x}$ = const, $\bar{y}$ = const, $\bar{z}$ = const).

The fulfillment of these similarity conditions was tested experimentally. As the models we used plates whose cross sections were made in the form of a right angle with a side L while the longitudinal dimensions were greater than the maximum transverse dimension of the jet. The investigations were conducted in strongly underexpanded air jets ($\gamma_j = 1.4$) discharging from sonic nozzles ($M_j = 1$). The pressure distribution over the surface of the plate was measured in the experiments.

The results of these investigations are presented in Fig. 3 with $K_1 = 0.115$ and $K_2 = 94$. The distribution of the pressure $\bar{p}_w = p_w/p_\infty$ at the surface of the plate in three jet cross sections $\bar{x} = 0.29$, 0.58, and 0.86 as a function of $\bar{y}$ is given in it (data a, b, and c). The experimental points in this figure correspond to the following values of the parameters: 1) $p_{0j} = 440$ mm Hg, $p_\infty = 0.027$ mm Hg, L = 40 mm, $d_j = 1.89$ mm, $T_{0j} = 295$°K;

2) $p_{0j} = 220$ mm Hg, $p_\infty = 0.027$ mm Hg, $L = 20$ mm, $d_j = 1.89$ mm, $T_{0j} = 295$°K; 3) $p_{0j} = 31$ mm Hg, $p_\infty = 0.104$ mm Hg, $L = 20$ mm, $d_j = 10.1$ mm, $T_{0j} = 900$°K; 4) $p_{0j} = 59.4$ mm Hg, $p_\infty = 0.104$ mm Hg, $L = 20$ mm, $d_j = 7.3$ mm, $T_{0j} = 900$°K.

The experiment encompasses a wide range of variation of the parameters p_{0j}, p_∞, T_{0j}, L, and d_j and confirms the validity of the similarity law of [1]. The measured pressure at the plate is represented here in a unified way in the variables (1.3), since its value in the hypersonic region of the jet is proportional to the velocity head ρU^2, for which the similarity in these variables is retained in the entire field of flow except for the vicinity of the nozzle exit cross section.

In the general case such a representation of the results will always be possible in the mode of hypersonic stabilization for those regions of the jet in which $M \to \infty$. In this case the flow around a body depends only on the density and velocity in the undisturbed stream, i.e., on those variables for which the similarity in the variables (1.3) is retained in the entire field of flow of the jet.

Upon violation of the principle of independence of the flow from the Mach number, such as with sufficiently small angles τ between the velocity vector and the surface of the streamlined body, the similarity in the variables (1.3) in the hypersonic region of the jet at finite values of the pressure drop p_∞/p_{0j} will be violated; the more so, the smaller the hypersonic similarity parameter $M\tau$ becomes.

In Fig. 4 this is illustrated on the example of flow over a rectangular plate with a characteristic size L and a relative thickness $t/L = 0.05$. In it the variation of the tangential and normal forces $\bar{T} = T/p_\infty L^2$ and $\bar{N} = N/p_\infty L^2$ acting on the plate as a function of $\bar{x}$ along the axis of a strongly underexpanded air jet ($\gamma_j = 1.4$) discharging from a sonic nozzle ($M_j = 1$) is represented in the variables (1.3) for values of the angle of attack of $\alpha = 0$, 5, 10, 20, and 30° (curves 1-5, respectively). The investigations were carried out with $K_1 = 0.055$, $K_2 = 90$, and $T_W/T_{0j} = 1$; the experimental points in the figure correspond to the following values of the parameters: 1) $p_{0j} = 440$ mm Hg, $p_\infty = 0.012$ mm Hg, $L = 20$ mm, $d_j = 1.89$ mm, $T_{0j} = 295$°K; 2) $p_{0j} = 27$ mm Hg, $p_\infty = 0.024$ mm Hg, $L = 10$ mm, $d_j = 5.4$ mm, $T_{0j} = 295$°K. The difference in these functions at $\bar{x} < \bar{x}_+$, which increases as the angle of attack decreases, is clearly seen.

In this case in the interaction of the hypersonic region of a jet with bodies the system of similarity criteria and of the dimensionless functions and independent variables should be written in the form (1.4) and supplemented by the parameters L/d_j and T_W/T_{0j}.

The results of the corresponding experimental studies are presented in Fig. 5. In it, as in the preceding case, the variation of the tangential and normal forces $T' = T/p_{0j}L^2$ and $N' = N/p_{0j}L^2$ acting on a rectangular plate as a function of x' along the axis of a strongly underexpanded air jet ($\gamma_j = 1.4$) discharging from a sonic nozzle ($M_j = 1$) is presented for the same values of the angle of attack α but now in the variables (1.4)($Re_j = 1044$, $L/d_j = 1$, $T_W/T_{0j} = 1$). The experimental points in the figure correspond to the following values of the parameters: 1) $p_{0j} = 2.59$ mm Hg, $p_\infty = 0.007$ mm Hg, $d_j = 20$ mm, $T_{0j} = 295$°K; 2) $p_{0j} = 5.2$ mm Hg, $p_\infty = 0.007$ mm Hg, $d_j = 10$ mm, $T_{0j} = 295$°K.

It is obvious that similarity is observed in these variables in the hypersonic region of the jet.

Attention should be turned to the fact that with a decrease in the temperature factor T_W/T_{0j} from 1 to 0.5 (Fig. 3) the criterial functions constructed in similarity parameters vary slowly within the limits of the experimental accuracy. This does not contradict the conclusion established earlier (see [11], for example) that the temperature factor has a slight effect on the aerodynamic characteristics of bodies when a rarefied gas stream flows over them in the mode of hypersonic stabilization.

LITERATURE CITED

1. V. N. Gusev and V. V. Mikhailov, "On the similarity of flows with expanding jets," Uch. Zap. Tsentr. Aerogidrodin. Inst., 1, No. 4 (1970).
2. V. N. Gusev and A. V. Zhbakova, "The flow of a viscous heat-conducting compressible fluid into a constant pressure medium," Rarefied Gas Dynamics, Vol. 1, Academic Press, New York—London (1969).
3. M. D. Ladyzhenskii, "An analysis of equations of hypersonic flows and the solution of the Cauchy problem," Prikl. Mat. Mekh., 26, No. 2 (1962).
4. V. N. Gusev and A. V. Zhbakova, "Properties of the spherical expansion of a viscous gas into a flooded space," Uch. Zap. Tsentr. Aerogidrodin. Inst., 7, No. 4 (1976).
5. N. C. Freeman and S. Kumar, "On the solution of the Navier—Stokes equations for a spherically symmetric expanding flow," J. Fluid Mech., 56, Part 3 (1972).

6. E. P. Muntz, B. B. Hamel, and B. L. Maguire, "Some characteristics of exhaust plume rarefaction," AIAA J., 8, No. 9 (1970).
7. V. N. Gusev, " On the influence of viscosity in jet flows," Uch. Zap. Tsentr. Aerogidrodin. Inst., 1, No. 6 (1970).
8. V. S. Avduevskii, A. V. Ivanov, I. M. Karpman, V. D. Traskovskii, and M. Ya. Yudelovich, "Flow in a supersonic, viscous, underexpanded jet," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1970).
9. V. S. Avduevskii, A. V. Ivanov, I. M. Karpman, V. D. Traskovskii, and M. Ya. Yudelovich, "Influence of viscosity on the flow in the initial section of a strongly underexpanded jet," Dokl. Akad. Nauk SSSR, 197, No. 1 (1971).
10. V. V. Volchkov, A. V. Ivanov, N. I. Kislyakov, A. K. Rebrov, V. A. Sukhnev, and R. G. Sharafutdinov, "Low-density jets beyond a sonic nozzle at large pressure drops," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1973).
11. V. N. Gusev, A. I. Erofeev, T. V. Klimova, V. A. Perepukhov, V. V. Ryabov, and A. I. Tolstykh, "Theoretical and experimental investigations of flow over bodies of simple shape by a hypersonic stream of rarefied gas," Tr. Tsentr. Aerogidrodin. Inst., No. 1855 (1977).

NUMERICAL INVESTIGATION OF GASDYNAMIC FEATURES OF CONTROL JETS

V. M. Dvoretskii and V. V. Zelentsov

UDC 533.6.011.35:518.5

Numerical methods, based on first order difference schemes are used to investigate features of three-dimensional subsonic and supersonic flows of an inviscid non-heat-conducting gas in control jets. Elements of the nozzle channels considered are axisymmetric, and flow symmetry arises from the nonaxial feature of the prenozzle volume and the subsonic part of the nozzle, or because of nonaxiality of elements of the supersonic part. In the first case the nozzle includes an asymmetric subsonic region in which reverse-circulatory flow is observed, and in the second case it includes a region of sudden expansion of the supersonic flow from the asymmetric stagnation zone.

A number of features of supersonic three-dimensional gas flows in nozzles of various shapes have been investigated using small perturbation theory and the method of characteristics, and by numerical integration of the system of gasdynamic equations including also analysis of mixed flows over the entire channel of an asymmetric nozzle (a bibliography of numerous investigations is included, e.g., in [1, 2]). An experimental determination mainly of integral characteristics of asymmetric nozzles, e.g., of the lateral stress, has confirmed the laws predicted by theoretical investigations ([3] and the bibliography there). However, most of the published papers contain an analysis of three-dimensional flows in nozzle channels of rather simple form. In the general class of control nozzles the shape of the channel is substantially complex. By investigating the local flow structure in such nozzles one can justify choice of component elements and can formulate laws for the shaping of control forces. The present paper attempts, in the perfect gas approximation, to analyze features of the distribution of local and integral characteristics of flow asymmetry in a hinged nozzle where the axis of rotation of the moving part lies in the throat region, and the subsonic part is recessed in the adjacent cylinder in such a way that the joint section between them is an asymmetric region (a "slot"), whose shape in an arbitrary meridian section is described by a two-valued function. Another object of investigation is a truncated nozzle. There is a discontinuity in contour between the initial section of the supersonic funnel and the fixed end part, and this causes a sudden flow expansion, with formation of special features in the subsequent flow region.

A three-dimensional variant of the Godunov scheme [4] is used to calculate the flow in the subsonic and transonic parts of the hinged nozzle by a time-dependent method. The supersonic flow calculations are carried out using the steady-state analog of this scheme [5]. It has been shown that in the recessed hinged nozzle there is a complex spiral-shaped flow with reverse flow zones. The investigation conducted shows that it is possible

Moscow. Translated from Izvestiya Akademii Nauk SSSR, Mekhanika Zhidkosti i Gaza, No. 6, pp. 126-133, November-December, 1978. Original article submitted November 28, 1977.

