10. J. P. Parker, "Rotational and vibrational relaxation in diatomic gases," Phys. Fluids, 2, No. 4 (1959).
11. C. F. Hansen and W. E. Pearson, "Three-dimensional model of collision-induced vibrational transitions in homonuclear diatomic molecules," J. Chem. Phys., 53, No. 9 (1970).
12. E. H. Carnevale, C. Carey, and G. Larson, "Ultrasonic determination of rotational collision numbers and vibrational relaxation times of polyatomic gases at high temperatures," J. Chem. Phys., 47, No. 8 (1967).

SPHERICAL EXPANSION OF A BINARY GAS MIXTURE INTO A FLOODED SPACE

V. N. Gusev and V. V. Ryabov

UDC 533.6.011.8:532.525.2

INTRODUTION

An increasing interest in the study of jet flows in connection with their possible use in the separation of isotopes or gas mixtures has recently been noted. Direct observations of the separation of mixtures during the penetration of the ambient gas into the jet (see [1], for example) serve as the basis for this.

One-dimensional flow from a source can become a good theoretical model for the study of separation processes in jet flows. The properties of the spherical expansion of a viscous heat-conducting gas into a flooded space have been studied in detail on the example of this flow using the Navier–Stokes equations (see the bibliography of [2]). In the investigation of separation effects one must allow for diffusional processes in addition to viscosity and heat conduction. In theoretical investigations great attention has been paid to this question in the study of the structure of a plane shock wave in a binary gas mixture. The present investigation was undertaken for the purpose of clarifying the role of diffusional processes in one-dimensional flow from a spherical source. The spherical shock wave, in which the complete separation of the components of a mixture is possible in the presence of a small counterpressure, as shown in the report, is studied in detail. Asymptotic solutions are obtained in the transonic and hypersonic regions of flow. The results of numerical calculations are presented for argon–helium mixtures at different initial concentrations.

§1. Let us consider the established supersonic flow of a gas mixture escaping from a spherical source of radius n_* with the velocity of sound (Mach number $M_* = 1$) into a space with a constant pressure p_∞. The flow will occur along radii from points of the sphere with the center at the origin of coordinates and will consist of two regions, inner supersonic ($r_* < r < r_+$) and outer subsonic, separated by some transitional region. In the case of an ideal gas the flow is described by the Euler equations and r_+ is the coordinate of the shock wave; in the presence of viscosity r_+ is the coordinate where the flow parameters are extremal.

Following [3], we write the system of one-dimensional Navier–Stokes equations for a one-temperature gas mixture in the case of spherical symmetry:

$$\begin{gathered}
\rho u r^2=\rho_1 u_1 r^2+\rho_2 u_2 r^2=\text{const},\qquad \rho_1 u_1 r^2=\text{const},\qquad p=\frac{R_0}{m}\rho T\\
\rho u\frac{du}{dr}+\frac{dp}{dr}=\frac{4}{3}\mu\left[\frac{d^2u}{dr^2}+\frac{2}{r}\frac{du}{dr}-\frac{2u}{r^2}\right]+\frac{4}{3}\frac{d\mu}{dr}\left[\frac{du}{dr}-\frac{u}{r}\right]\\
\rho u r^2\left(c_pT+\frac{u^2}{2}\right)-r^2\lambda\frac{dT}{dr}+\rho_1 r^2(u_1-u)\times\\
\times\left(c_{p1}T-c_{p2}T+\beta\frac{p}{\rho}\frac{m^2}{m_1m_2}\right)-\frac{4}{3}r^2\mu u\left(\frac{du}{dr}-\frac{u}{r}\right)=\text{const}\\
u_1-u=\frac{\rho}{\rho_1}\frac{m_1m_2}{m^2}D_{12}\left\{f(1-f)\left[\frac{m_1-m_2}{m}\frac{d}{dr}(\ln p)-\beta\frac{d}{dr}(\ln T)\right]-\frac{df}{dr}\right\}
\end{gathered}\tag{1.1}$$

Moscow. Translated from Izvestiya Akademii Nauk SSSR, Mekhanika Zhidkosti i Gaza, No. 2, pp. 108-117, March-April, 1978. Original article submitted May 5, 1977.

Here u is the velocity; p, ρ, and T are the pressure, density, and temperature, respectively; f is the concentration; m is the molecular weight; R_0 is the universal gas constant; μ is the coefficient of viscosity; λ is the coefficient of thermal conductivity; c_p is the specific heat at constant pressure; D_{12} is the coefficient of diffusion; β is the thermal diffusion ratio; the component with the heavier molecular weight is denoted by the index 1 and the lighter one by the index 2.

The average density of the gas and its molecular weight and specific heat at constant pressure are defined as

$$\rho=\rho_1+\rho_2, \quad m=m_1 f+m_2(1-f), \quad \left(f=\frac{\rho_1 m_1}{\rho m}\right),$$

$$c_p=\frac{\gamma R_0}{m(\gamma-1)}=\frac{(c_{p1}\rho_1+c_{p2}\rho_2)}{\rho}$$

Here γ is the ratio of specific heats. We also assume that $\mu \sim T^n$, $\beta \sim m^{-1}$, the Prandtl number $\sigma = \mu c_p/\lambda \sim m$, and the Schmidt number $Sc = \mu/(\rho D_{12}) \sim m^{-1}$. With n = 0.75 these functions well approximate the transfer coefficients of an argon–helium mixture [3].

Let us reduce the system (1.1) to dimensionless form. We introduce the following dimensional constants: $Q = Q_1 + Q_2 = 4\pi\rho u r^2$ is the gas flow rate; p_∞, ρ_∞, T_∞, and μ_∞ are the pressure, density, temperature, and coefficient of viscosity at $r = \infty$. The dimensionless quantities are defined as follows:

$$w=\frac{u}{\sqrt{\gamma R_0 T_\infty/m_\infty}}, \quad p'=\frac{p}{p_\infty}, \quad \rho'=\frac{\rho}{\rho_\infty} \quad \theta=\frac{T}{T_\infty}$$

$$\frac{\mu}{\mu_\infty}=\theta^n, \quad \frac{\sigma}{\sigma_\infty}=\frac{F}{F_\infty}, \quad \frac{\mathrm{Sc}}{\mathrm{Sc}_\infty}=\frac{\beta}{\beta_\infty}=\frac{F_\infty}{F} \tag{1.2}$$

$$y=\frac{l}{r}=\left[\frac{Q\sqrt{\gamma R_\infty T_\infty/m_\infty}}{4\pi\gamma p_\infty}\right]^{0.5} r^{-1}, \quad F=\varepsilon+(1-\varepsilon)f, \quad \varepsilon=\frac{m_2}{m_1}$$

Here the quantities at an infinitely remote point $r = \infty$ are denoted by the index ∞.

After the pressure and density are eliminated the system (1.1) is reduced to the following form:

$$\frac{dw}{dy}+\frac{1}{\gamma}\frac{F_\infty}{F}\left[\frac{1}{\theta}\frac{d\theta}{dy}-\frac{1}{w}\frac{dw}{dy}+\frac{2}{y}-\frac{(1-\varepsilon)}{F}\frac{df}{dy}\right]\frac{\theta}{w}=-\frac{1}{C}\left[\theta^n\left(\frac{d^2w}{dy^2}-\frac{2w}{y^2}\right)+n\theta^{n-1}\frac{d\theta}{dy}\left(\frac{dw}{dy}+\frac{w}{y}\right)\right]$$

$$\left[\frac{Q_1}{Q}+\left(1-\frac{Q_1}{Q}\right)\varepsilon^{-1}\right]F_\infty\theta+(\gamma-1)w^2\left(0.5+\frac{\theta^n}{(yC)}\right)+\frac{\theta^n}{C}\left[0.75F_\infty\Big/(\sigma_\infty F)\frac{d\theta}{dy}+(\gamma-1)w\frac{dw}{dy}\right]+$$

$$+0.5(\gamma-1)\beta_\infty F_\infty^2\theta\left(\frac{Q_1}{Q}-\frac{f}{F}\right)\varepsilon^{-1}=\alpha \tag{1.3}$$

$$\frac{Q_1}{Q}-\frac{f}{F}=\frac{3\varepsilon}{4\,\mathrm{Sc}_\infty F F_\infty}\frac{\theta^n}{C}\left\{f(1-f)\left[\left(\frac{1}{w}\frac{dw}{dy}-\frac{1}{\theta}\frac{d\theta}{dy}-\right.\right.\right.$$

$$\left.\left.\left.-\frac{2}{y}\right)\frac{1-\varepsilon}{F}+\beta_\infty\frac{F_\infty}{F}\frac{1}{\theta}\frac{d\theta}{dy}\right]+\frac{\varepsilon^2+(1-\varepsilon^2)f}{F^2}\frac{df}{dy}\right\}$$

Here α and Q_1/Q are dimensionless constants determining the fluxes of heat and diffusion passing through a spherical surface per unit time at an infinitely remote point $r = \infty$ $(y = 0)$, which must be assigned as the boundary conditions.

In the case when the pressure approaches a finite value the solution of the system (1.2) in the vicinity of the point y = 0 has the following form:

$$w=\sum_{j=0}^{\infty}a_j y^{j+2}, \quad \theta=\sum_{j=0}^{\infty}b_j y^j, \quad f=\sum_{j=0}^{\infty}c_j y^j$$

From the boundary conditions for the flow rate of the gas and its temperature and concentration at $y = 0$ it follows that $a_0 = b_0 = 1$ and $c_0 = f_\infty$. The subsequent coefficients of the series are determined through the dimensionless constants C, Q_1/Q, Sc_∞, α, γ, n, σ_∞, β_∞, f_∞, and ε.

For example,

$$a_1 = b_1 - c_1(1-\varepsilon)/F_\infty$$

$$b_1 = \frac{4}{3}\sigma_\infty(\gamma-1)\frac{C}{\gamma}\left\{\frac{\gamma\alpha}{\gamma-1} - \frac{\gamma F_\infty}{\gamma-1}\left[\frac{Q_1}{Q} + \left(1-\frac{Q_1}{Q}\right)\frac{1}{\varepsilon}\right] - \frac{\beta_\infty F_\infty^2}{\varepsilon}\left(\frac{Q_1}{Q} - \frac{f_\infty}{F_\infty}\right)\right\}$$

$$c_1 = \frac{4}{3}Sc_\infty\frac{F_\infty^2 C}{\varepsilon}\left(\frac{Q_1}{Q} - \frac{f_\infty}{F_\infty}\right) - f_\infty(1-f_\infty)\beta_\infty b_1$$

In the case when thermal and diffusional fluxes at an infinitely remote point are absent ($b_1 = c_1 = 0$), we have $\alpha = 1$ and $Q_1/Q = f_\infty/F_\infty$.

§2. Let us dwell on this case in more detail. We will consider an argon–helium mixture ($\gamma = 1.67$, $n = 0.75$, $\varepsilon = 0.1$).

The system of equations (1.3) was solved numberically on a computer by the method presented in [4]. The solution has much in common with the case of the escape of a one-component gas. At relatively high Reynolds numbers $Re_* = \rho_* u_* r_* / \mu_*$, where the asterisk denotes quantities at the critical sphere, the flow remains close to ideal at $r < r_+$.

The results of the numberical calculations with $f_\infty = 0.5$, $Sc_\infty = 0.333$, $\sigma_\infty = 0.431$, and $\beta_\infty = 0.377$ are presented in Fig. 1. In it the variations in the pressure p' and density ρ' of the mixture are given in similarity variables [5] as functions of $x' = r_*/[r(p_{0*}/p_\infty)^{0.5}]$ at values of the criterion $C = 5$ (curves 1) and 0.07 (curves 2); C is uniquely connected with the similarity parameter $K_2 = Re_*(p_\infty/p_{0*})^{0.5}$ [5]:

$$C = 0.75(0.5(\gamma+1))^{[0.25(\gamma+1)/(\gamma-1)-n]}\left(\frac{m_\infty}{m_*}\right)^{0.25}\left(\frac{T_{0*}}{T_\infty}\right)^{n+0.25}K_2$$

where the stagnation parameters are denoted by the index 0. As shown by the results obtained, the variations in the parameters of the mixture at $r > r_+$ are analogous to the corresponding variations in a one-component gas [2].

In the presence of dissipative processes the spherical shock wave becomes smeared out and its thickness becomes finite and the greater, the larger the pressure drop p_{0*}/p_∞. In contrast to a plane shock wave, because of the spreading out of the gas the ratios at its front, which follow from the theory of a direct compression shock, are disrupted. Moreover, as $p_{0*}/p_\infty \to \infty$, when $C \to 0$, the density drop at the shock wave front disappears entirely and the density variation becomes monotonic in the entire region of flow (curve 2 in Fig. 1).

Because of the large gradients of the thermodynamic quantities in the shock wave front, diffusion fluxes of the components of the mixture develop, thanks to which their redistribution occurs. The considerable increase in the velocity of the light helium component in this region attracts attention. In the example under consideration its maximum velocity exceeds the limiting velocity of the mixture by more than three times.

Concentration of the light component occurs in the leading front of the spherical shock wave, just as in the plane case [6]. This effect was noted in [7]. Its magnitude depends on the initial concentration of the mixture and is practically constant with variation in the similarity parameter K_2 (see Fig. 2, in which curves 1-4 represent the results of the calculation of the argon concentration for $K_2 = 0.087$, 1.24, 3.72, and 12.4, respectively). At small values of f_∞ the concentration of the light component becomes considerable; for example, at $f_\infty = 0.02$ and $K_2 = 1.24$ the minimum value is $f = 0.5f_\infty$.

The variation in the concentration f at $r > r_+$ presented in Fig. 2 shows that as $K_2 \to 0$, when the drop $p_{0*}/p_\infty \to \infty$, an ever increasing enrichment of the mixture with the heavy component occurs inside the shock wave front owing to the concentration of the light component. The solution of the system (1.3) corresponding to this region as $C \to 0$ has the form

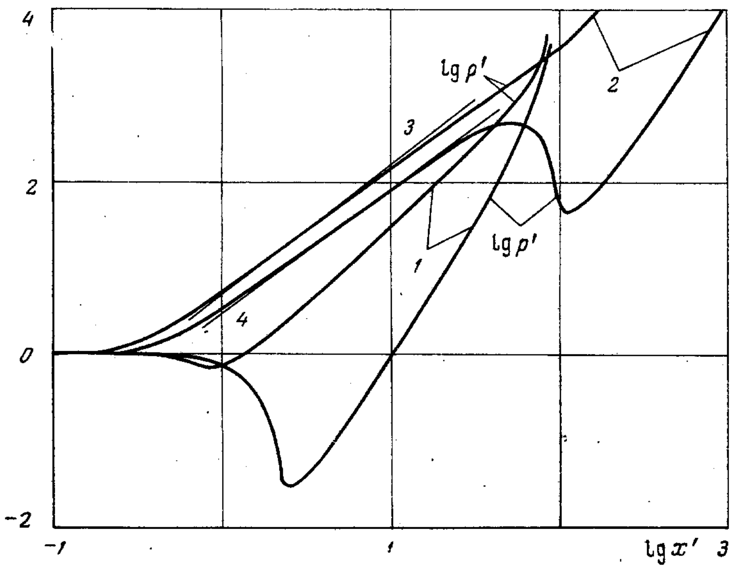

Fig. 1

$$w=\sum_{j=0}^{\infty} g_j y^{j+1/2},\qquad \theta=\sum_{j=0}^{\infty} d_j y^j,\qquad f=\sum_{j=0}^{\infty} h_j y^j \tag{2.1}$$

From the diffusion equation it follows that the coefficient $h_0 = 1$, i.e., with $p_{0*}/p_\infty \gg 1$ the complete separation of the components of the mixture ($f \to 1$ as $y \to 0$) occurs in a spherical shock wave.

The subsequent coefficients of the series (2.1) are expressed through d_0; for example, $g_0{}^2 = {}^2/_3(c/\gamma)F_\infty d_0{}^{1-n}$. Being confined henceforth to the first terms of the expansions (2.1), for the flow parameters in this region we can write

$$w^2=0.5\left(\frac{2}{\gamma+1}\frac{T_{0*}}{T_\infty}\right)^n d_0^{1-n}F_\infty K_2\frac{x'}{\gamma},\qquad p'=d_0F_\infty\rho'$$

$$\rho'^2=2\gamma m_*\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)-n}\left(\frac{T_\infty}{T_{0*}}\right)^{1+n}\frac{x'^3}{(m_\infty d_0{}^{1-n}F_\infty K_2)}$$

A comparison of the equations obtained for the density and pressure with the results of numerical calculations with C = 0.07 is presented in Fig. 1 (curves 3 and 4).

§3. Let us proceed to an analysis of the flow in the inner supersonic region ($r < r_+$). Here the flow remains close to ideal. The departures become appreciable in the regions adjacent to the critical section and to the leading front of the shock wave.

Here it is more convenient to conduct the analysis in new variables, made dimensionless with respect to their values in the critical section of the ideal source, henceforth denoted by the index *i. Then in place of the system (1.3) we will have

$$\begin{gathered}\frac{dv}{dx}+\frac{1}{\gamma}\frac{F_{*i}}{F}\left[\frac{1}{t}\frac{dt}{dx}-\frac{1}{v}\frac{dv}{dx}+\frac{2}{x}-\frac{(1-\varepsilon)}{F}\frac{df}{dx}\right]\frac{t}{v}=\\=-\frac{1}{R}\left[t^n\left(\frac{d^2v}{dx^2}-2\frac{v}{x^2}\right)+nt^{n-1}\frac{dt}{dx}\left(\frac{dv}{dx}+\frac{v}{x}\right)\right]\\t+(\gamma-1)v^2\left(0.5+\frac{t^n}{xR}\right)+\frac{t^n}{R}\left[\frac{3}{4\sigma_{*i}}\frac{F_{*i}}{F}\frac{dt}{dx}+\right.\\\left.+(\gamma-1)v\frac{dv}{dx}\right]+0.5(\gamma-1)\beta_{*i}\frac{F_{*i}{}^2}{\varepsilon}\left(\frac{f_{*i}}{F_{*i}}-\frac{f}{F}\right)t=0.5(\gamma+1)\\\frac{f_{*i}}{F_{*i}}-\frac{f}{F}=\frac{3\varepsilon}{4\,\mathrm{Sc}_{*i}F_{*i}F}\frac{t^n}{R}\left\{f(1-f)\left[\left(\frac{1}{v}\frac{dv}{dx}-\right.\right.\right.\\\left.\left.\left.-\frac{1}{t}\frac{dt}{dx}-\frac{2}{x}\right)\frac{1-\varepsilon}{F}+\beta_{*i}\frac{F_{*i}}{F}\frac{1}{t}\frac{dt}{dx}\right]+\frac{\varepsilon^2+(1-\varepsilon^2)f}{F^2}\frac{df}{dx}\right\}\end{gathered} \tag{3.1}$$

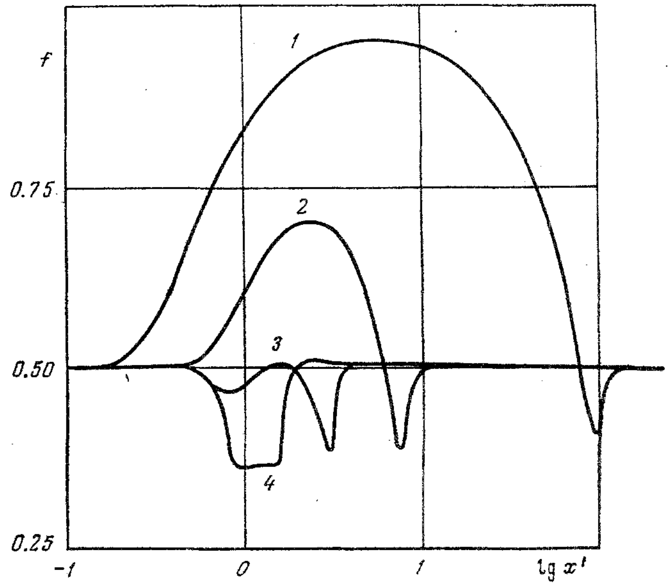

Fig. 2

$$x=\frac{r_{*i}}{r},\quad v=u\left(\frac{\gamma R_0 T_{*i}}{m_{*i}}\right)^{-0.5}\qquad t=\frac{T}{T_{*i}}$$

$$R=0.75\mathrm{Re}_{*i}=0.75\rho_{*i}u_{*i}r_{*i}/\mu_{*i}$$

In the case of an ideal gas the system (3.1) has the solution

$$x^2=v[0.5(\gamma+1)-0.5(\gamma-1)v^2]^{1/(\gamma-1)},\qquad t=0.5(\gamma+1)-0.5(\gamma-1)v^2,\ f=f_{*i} \tag{3.2}$$

which, in the transonic region in the vicinity of the point x = 1, can be represented in the form

$$v=1+\frac{2}{\sqrt{\gamma+1}}(1-x)^{0.5}+\ldots,\qquad t=1-\frac{2(\gamma-1)}{\sqrt{\gamma+1}}(1-x)^{0.5}+\ldots \tag{3.3}$$

To construct the asymptotic solution in this region in the case of a binary mixture we use the method of deformable coordinates. Such an approach was used in [8] for a one-component gas with μ = const. Converting to the new dependent and independent variables

$$V=R^{1/3}(v-1),\quad G=R^{1/3}(1-t),\quad \Phi=R^{2/3}(f-f_{*i}),\quad X=R^{2/3}(1-x) \tag{3.4}$$

in the system (3.1) and taking $R\to\infty$, we obtain

$$A\frac{d^2V}{dX^2}-(\gamma+1)V\frac{dV}{dX}+2=0,\qquad G=(\gamma-1)V$$

$$\Phi=\frac{0.75f_{*i}(1-f_{*i})}{\mathrm{Sc}_{*i}}\left[\frac{1-\varepsilon}{F_{*i}}\gamma-\beta_{*i}(\gamma-1)\right]\frac{dV}{dX} \tag{3.5}$$

$$A=1+0.75\frac{(\gamma-1)}{\sigma_{*i}}+0.75\frac{f_{*i}(1-f_{*i})}{\mathrm{Sc}_{*i}}\left(\frac{1-\varepsilon}{F_{*i}}-\frac{\gamma-1}{2}\beta_{*i}\right)\left[\frac{1-\varepsilon}{F_{*i}}\gamma-\beta_{*i}(\gamma-1)\right]$$

Integrating the first equation of the system (3.5), we obtain

$$A\frac{dV}{dX}-\frac{\gamma+1}{2}V^2+2X=0$$

Its solution is expressed through Bessel functions:

$$V=2\sqrt{\frac{X}{\gamma+1}}\frac{I_{-2/3}(\delta)-I_{2/3}(\delta)}{I_{-1/3}(\delta)-I_{1/3}(\delta)}\qquad (X>0),$$

$$V=2\sqrt{\frac{|X|}{\gamma+1}}\frac{J_{-2/3}(\delta)-J_{2/3}(\delta)}{J_{-1/3}(\delta)+J_{1/3}(\delta)}\qquad (X<0)$$

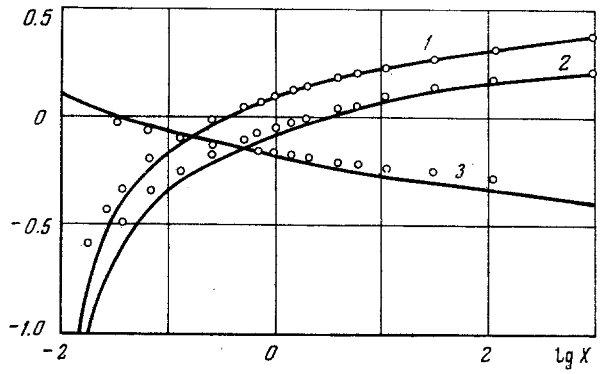

Fig. 3

$$\delta=\frac{2\sqrt{\gamma+1}}{3A}|X|$$

$$V=2\sqrt{\frac{X}{\gamma+1}}\left(1+\frac{0.25A}{\sqrt{\gamma+1}}\frac{1}{X^{2/3}}+\dots\right)\qquad (X\to\infty) \tag{3.6}$$

In this case the solution (3.6) asymptotically approaches the solution (3.3) for an ideal gas. As $X \to -\infty$ the region of definition of the solution will be bounded by a limiting line, just as in the case of a one-component gas [4].

The velocity of the mixture at the point r_{*i} will be

$$v_{*i}=1+V(0)R^{-1/3},\qquad V(0)=\frac{2}{\sqrt{\gamma+1}}\left[\frac{3A}{\sqrt{\gamma+1}}\right]^{1/3}\frac{\Gamma(2/3)}{\Gamma(1/3)} \tag{3.7}$$

Through a comparison of the numerical solution obtained in Part 2 with the asymptotic solution (3.6) the coordinate of the critical section of the ideal source in the variables (1.2) is determined from Eq. (3.7). In fact, with $\alpha = 1$ we have

$$T_\infty=0.5(\gamma+1)T_{*i},\quad f_\infty=f_{*i},\quad v=\sqrt{0.5(\gamma+1)}\,w,\quad R=y_{*i}(0.5(\gamma+1))^nC$$

and to determine the coordinate y_{*i} from Eq. (3.7) we will have

$$\sqrt{0.5(\gamma+1)}\,w_{*i}=1+V(0)\,[y_{*i}(0.5(\gamma+1))^nC]^{-1/3}$$

In this case the transfer coefficients σ_{*i}, Sc_{*i}, and β_{*i} entering into the expressions presented above will coincide with the corresponding values at infinity.

The asymptotic solution (3.6) for an argon–helium mixture (lines) is compared with the numerical calculations with $C = 0.07$, $f_\infty = 0.5$, $Sc_\infty = 0.333$, $\sigma_\infty = 0.431$, and $\beta_\infty = 0.377$ (points) in Fig. 3 (1-3 correspond to log V, log G, and log Φ). Despite the relatively low value $Re_* = 915$, the agreement is fully satisfactory.

In the section $r = r_*$, where $M_* = 1$, the concentration f_* of the mixture increases with a decrease in Re_* while the velocities of the components are $u_{*1} < u_* < u_{*2}$. The increase in the velocity of the light component becomes considerable. For example, at $Re_* = 10$ it exceeded the critical velocity of the mixture by more than two times.

Let us turn to a consideration of the hypersonic region of flow. In the case of an ideal gas the expansions

$$v=\sqrt{\frac{(\gamma+1)}{(\gamma-1)}}-\frac{1}{\sqrt{\gamma^2-1}}\left[\frac{(\gamma-1)}{(\gamma+1)}\right]^{1/2(\gamma-1)}x^{2(\gamma-1)}+\dots,$$
$$t=\left[\frac{(\gamma-1)}{(\gamma+1)}\right]^{1/2(\gamma-1)}x^{2(\gamma-1)}+\dots \tag{3.8}$$

which follow from (3.2) are valid for the flow parameters in it.

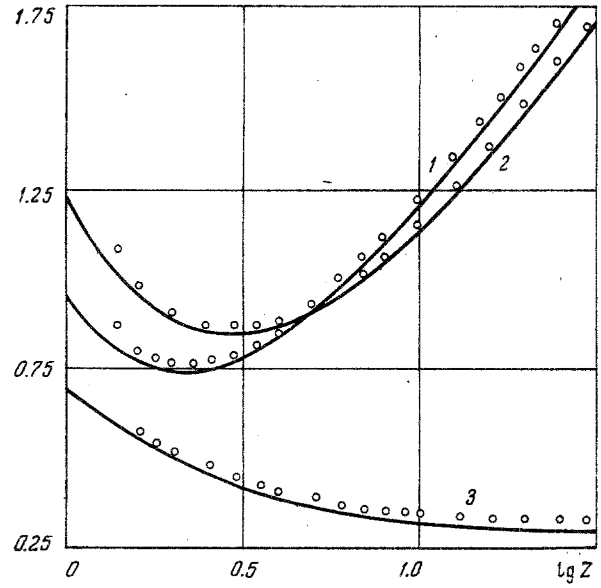

Fig. 4

To construct the asymptotic solution of the system (3.1) in the hypersonic region of flow we change to new variables in it:

$$W=(v-\sqrt{(\gamma+1)/(\gamma-1)})R^{\lambda},\ \Theta=tR^{\lambda},\ \Psi=(f-f_{*i})R^{\lambda}$$
$$Z=xR^{\omega},\ \omega=[2\gamma-1-2(\gamma-1)n]^{-1},\ \lambda=2\omega(\gamma-1)$$

After the substitution, and taking $R \to \infty$, we obtain

$$\frac{dW}{dZ}+nw_0\frac{\Theta^{n-1}}{Z}\frac{d\Theta}{dZ}-\frac{2w_0\Theta^n}{Z^2}=-\frac{1}{\gamma w_0}\left[\frac{d\Theta}{dZ}+\frac{2\Theta}{Z}\right]$$
$$\Theta+(\gamma-1)w_0\left(W+\frac{w_0}{Z}\Theta^n\right)=0,\qquad w_0^2=\frac{(\gamma+1)}{(\gamma-1)}$$
$$\Psi=\frac{0.75\varepsilon f_{*i}(1-f_{*i})}{\mathrm{Sc}_{*i}F_{*i}}\left(\frac{1-\varepsilon}{F_{*i}}-\beta_{*i}\right)\Theta^{n-1}\frac{d\Theta}{dZ}$$

The resulting system of equations was separated, and it contains the solution for a one-component gas [9]. Using it, with $n \neq 1$ we obtain

$$\Theta=\left\{\frac{w_0^2\gamma(\gamma-1)(1-n)}{[2\gamma-1-2n(\gamma-1)]Z}+\theta_1^{1-n}Z^{2(\gamma-1)(1-n)}\right\}^{1/(1-n)}$$
$$W=-\frac{\Theta}{w_0(\gamma-1)}-\frac{w_0\Theta^n}{Z},\qquad \theta_1=\left[\frac{(\gamma-1)}{(\gamma+1)}\right]^{1/2(\gamma-1)} \tag{3.9}$$
$$\Psi=\frac{0.75\varepsilon f_{*i}(1-f_{*i})}{\mathrm{Sc}_{*i}F_{*i}(1-n)}\left(\frac{1-\varepsilon}{F_{*i}}-\beta_{*i}\right)\left\{2(\gamma-1)(1-n)\theta_1^{1-n}Z^{2(\gamma-1)(1-n)-1}-\frac{w_0^2\gamma(\gamma-1)(1-n)}{[2\gamma-1-2n(\gamma-1)]Z^2}\right\}\Theta^{2n-1}$$

As $Z \to \infty$ the solution (3.9) asymptotically approaches the solution (3.8) for an ideal gas. The function Θ reaches its minimum value Θ_+ at a finite value $Z_+ > 0$, corresponding to the region of the leading front of a closing compression shock. The function $-W$ varies in a similar way. These properties for an argon–helium mixture are illustrated by Fig. 4, in which the asymptotic solution (3.9) (lines) is compared with the numerical calculations with $C = 0.07$, $f_\infty = 0.5$, $\mathrm{Sc}_\infty = 0.333$, $\sigma_\infty = 0.431$, and $\beta_\infty = 0.377$ (points); curves 1-3 correspond to $\log\Theta$, $\log(-W)$, and $\log\Psi$.

§4. Up to now the spherical expansion of a binary gas mixture into a flooded space has been analyzed with a zero diffusional flux at an infinitely remote point. In this case

$$\frac{Q_1}{Q}=\frac{f_\infty}{F_\infty}=\frac{fu_1}{(Fu)},\qquad \lim_{y\to 0}\left(\frac{u_1}{u}\right)=1$$

and, as the calculations show, the concentration in the critical section of a spherical source differs little from f_∞.

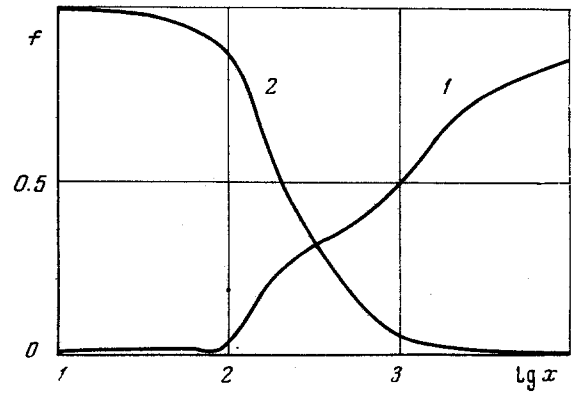

Fig. 5

The latter condition will be violated in the presence of a diffusional flux at an infinitely remote point. For example, for an argon–helium mixture with $f_\infty = 0.9$ and $Q_1/Q = 0.1$ we have $f_{*i} = 0.011$, and the case under consideration will correspond to the escape of helium with a slight argon content into a space filled by argon with a small admixture of helium. The distribution of the concentration f in such a flow with $Re_* = 453$ is presented in Fig. 5 (curve 1).

The case of the escape of argon with a slight helium content into a space filled by helium with a small admixture of argon can be analyzed in a similar way. With $f_\infty = 0.02$ and $Q_1/Q = 0.999$ we have $f_{*i} = 0.99$, and the distribution of the concentration f corresponding to this case with $Re_* = 78.5$ is presented in the same figure (curve 2).

The results of these calculations show that in both cases the gas of the surrounding space does not penetrate through the shock wave into the supersonic region of flow. The property indicated above was noted in [10] in the simplest case when the escaping and ambient gases were identical in their molecular properties.

LITERATURE CITED

1. E. P. Muntz and B. B. Hamel, "Rarefaction phenomena in gas and isotope separations," in: Rarefied Gas Dynamics, Proceedings of the Ninth International Symposium, Gottingen, 1974. Vol. 1, Porz-Wahn (1974).
2. V. N. Gusev and A. V. Zhbakova, "Properties of the spherical expansion of a viscous gas into a flooded space," Uch. Zap. Tsentr. Aéro-Gidrodinam. Inst., 7, No. 4 (1976).
3. S. Chapman and T. G. Cowling, The Mathematical Theory of Non-Uniform Gases, 2nd ed., Cambridge University Press, London–New York (1952).
4. V. N. Gusev and A. V. Zhbakova, "The flow of a viscous heat-conducting compressible fluid into a constant pressure medium," Rarefied Gas Dynamics, Vol. 1, New York–London (1969).
5. V. N. Gusev and V. V. Mikhailov, "On the similarity of flows in expanding jets," Uch. Zap. Tsentr. Aéro-Gidrodinam. Inst., 1, No. 4 (1976).
6. Ya. B. Zel'dovich and Yu. P. Raizer, The Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow (1966).
7. Yu. D. Nagornykh, "On the flow of a mixture of light and heavy gases behind a spherical shock wave," in: Gas Dynamics and Physical Kinetics [in Russian], Novosibirsk (1974).
8. A. Sakurai, "Three-dimensional, steady, radial flow of viscous, heat-conducting, compressible fluid," Quart. J. Mech. Appl. Math., 11, Part 3 (1958).
9. N. C. Freeman and S. Kumar, "On the solution of the Navier–Stokes equations for a spherically symmetric expanding flow," J. Fluid Mech., 56, Part 3 (1972).
10. P. A. Skovorodko and S. F. Chekmarev, "Gas diffusion in a low-density supersonic jet," in: Rarefied Gas Dynamics [in Russian], Novosibirsk (1976).