MOTIVATIONAL STUDY CASES FOR THE INTRODUCTORY MATHEMATICS COURSE IN COLLEGES

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Abstract

A set of new challenging motivational problems (with solutions) in Elementary Mathematics and Algebra is discussed. The problems are offered for students who are familiar with basic algebraic and geometrical concepts and are looking for modern applications of these concepts in advanced areas of mathematics (e.g., theories of numbers, logic, and algorithms) and computer science (e.g., encryption algorithms).

Introduction

The first course of Introductory Mathematics becomes a real challenge. Many our freshman students have a weak background in Mathematics. Some evening students took their last courses in high or middle school several years ago and forget even how to use simple arithmetic rules. Also there are advanced students, who are passionate for new knowledge, abstract mathematical concepts, and strong professional skills.

Traditional textbooks written for courses of Introductory Mathematics [1, 2] contain basic algebraic principles, simple algorithms, "drilling" exercises, trivial applications, and examples that are mostly oriented to "below-average-level" students. This gives no opportunity for advanced students to explore the beauty of real abstract mathematical concepts and modern applications of these concepts to advanced areas of mathematics, computer science, networking technologies, biology, sociology, business, and other disciplines.

A different approach should be developed and introduced for those students, who love explorations in mathematics and accept the challenges of modern applications of basic mathematical concepts. We would encourage students from local middle- and high-schools, college freshmen, and juniors to: (1) participate in regional, national, and (even) international contests in mathematics [3-9]; (2) take our specially-designed course, "Modern Applications of High-School Mathematics" in the Rivier College Challenge Program; (3) create the Students' Mathematics Club; (4) participate in extracurricular activities, such as a Mathematics Seminar; and (5) explore challenging study cases offered in traditional classes (e.g., Problem Solving and Modeling and Mathematics for the Humanities courses).

In the present paper, we introduce several non-trivial motivational study cases that have been discussed recently in our Introductory Mathematics courses and in the Challenge Program Mathematics course. It was a challenge for the instructor to find adequate problem prototypes in the literature. Popular mathematics journals, such as The College Mathematics Journal [10] and Mathematics Magazine [11] offer only advanced problems, which assume a high-level mathematical background of readers. Finally, the instructor has referred to the problems from the American Mathematics Competitions (available via

ISSN 1559-9388 (online version), ISSN 1559-9396 (CD-ROM version).

the Internet [4]) and unique books [8, 12] that he used in the past, when teaching Advanced Mathematics Seminars for high-school students in Russia. Here a set of motivational problems is offered for readers and Rivier College students. Solutions to selected problems are also discussed.

1. Study Case 1: Restoring the Digits in Arithmetic Calculations with Multiplication of Large Numbers

1.1 Problem: Restore digits in the example below, where the asterisks are placeholder for, possibly different, digits [12]:

$$\begin{array}{r} \ast\;1\;\ast\;\ast \\ \times\quad \underline{1\;\ast\;\ast} \\ \ast\;1\;\ast\;\ast \\ +\quad \ast\;\ast\;\ast\;1\;\ast \\ \underline{\ast\;\ast\;\ast\;1} \\ 8\;\ast\;\ast\;4\;\ast\;\ast \end{array} \qquad (1.1)$$

Find all solutions.

1.2 Solutions

We have to find all possible values for the parameters a, b, c, d, e, f, g, h, I, j, k, m, n, p, q, r, s, t, and x in the following expression:

$$\begin{array}{r} a\;1\;b\;c \\ \times\quad \underline{1\;d\;e} \\ f\;1\;g\;h \\ +\quad i\;j\;k\;1\;x \\ \underline{m\;n\;p\;1} \\ 8\;q\;r\;4\;s\;t \end{array} \qquad (1.2)$$

We can see that (mnp1) = (a1bc) × 1 = (a1bc). Therefore, m = a; n = 1; p = b; and c = 1.

Also, because the right most digit of e × 1 is 1, e must equal 1. As a result, we can find that f = a; g = b; h = c = 1, t =1, and x = d. After these substitutions into expression (1.2), we can find the following:

$$\begin{array}{r} a\;1\;b\;1 \\ \times\quad \underline{1\;d\;1} \\ a\;1\;b\;1 \\ +\quad i\;j\;k\;1\;d \\ \underline{a\;1\;b\;1} \\ 8\;q\;r\;4\;s\;1 \end{array} \qquad (1.3)$$

The right most digit of the product b × d must be 1. The digital parameter s can be estimated from the formulas:

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$$b + d = 10 + s; \quad b \times d = 10 \times w + 1 \tag{1.4}$$

There are only three cases that can satisfy the conditions of expression (1.4):

Case a): $b = 9$; $d = 9$, hence $b \times d = 81$; $b + d = 18$, and $s = 8$; (1.5a)

Case b): $b = 3$; $d = 7$, hence $b \times d = 21$; $b + d = 10$, and $s = 0$; (1.5b)

Case c): $b = 7$; $d = 3$, hence $b \times d = 21$; $b + d = 10$, and $s = 0$. (1.5c)

Let us study all these cases in detail.

1.2.1 Case (a)

Under the conditions (1.5a), we can find from expression (1.3) the following:

$$\begin{array}{rcccccc}
 & & & a & 1 & 9 & 1 \\
\times & & & & 1 & 9 & 1 \\ \hline
 & & & a & 1 & 9 & 1 \\
+ & i & j & k & 1 & 9 & \\
 & a & 1 & 9 & 1 & & \\ \hline
 & 8 & q & r & 4 & 8 & 1
\end{array} \tag{1.6a}$$

Here the parameter k must be equal to 7 only, and the following two relations must be satisfied between parameters a, i, and j:

$$10 \times i + j = 9 \times a + 1; \;\; a \geq 1; \;\; i > 0 \tag{1.7a}$$

$$7 \leq a + i \leq 8 \tag{1.8a}$$

These two conditions (1.7a, 1.8a) can be satisfied only for $a = 4$. Therefore, $i = 3$ and $j = 7$. Parameters r and q can be found by substituting all other known digits (a, i, j, and k) into expression (1.6a). We find $r = 0$ and $q = 0$. Finally, in this case, the expressions (1.1) and (1.6a) can be reduced to expression (1.9a) below:

$$\begin{array}{rr}
 & 4191 \\
\times & 191 \\ \hline
 & 4191 \\
+ & 37719 \\
 & 4191 \\ \hline
 & 800481
\end{array} \tag{1.9a}$$

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1.2.2 Case (b)

Under the conditions (1.5b), we can find from expression (1.3) the following:

$$\begin{array}{r} a\,1\,3\,1 \\ \times\quad 1\,7\,1 \\ \hline a\,1\,3\,1 \\ +\quad i\,j\,k\,1\,7 \\ a\,1\,3\,1 \\ \hline 8\,q\,r\,4\,0\,1 \end{array} \qquad (1.6b)$$

Here the parameter k must be equal to 9 only, and the following two relations must be satisfied between parameters a, i, and j:

$$10 \times i + j = 7 \times a;\;\; a \geq 2;\;\; i > 0 \qquad (1.7b)$$

$$7 \leq a + i \leq 8 \qquad (1.8b)$$

These two conditions (1.7b, 1.8b) can be satisfied only for a = 5; therefore, i = 3 and j = 5. Parameters r and q can be found by substituting all other known digits (a, i, j, and k) into expression (1.6b). We find r = 7 and q = 7. Finally, in this case, the expressions (1.1) and (1.6b) can be reduced to expression (1.9b) below:

$$\begin{array}{r} 5131 \\ \times\quad 171 \\ \hline 5131 \\ +\quad 35917 \\ 5131 \\ \hline 877401 \end{array} \qquad (1.9b)$$

1.2.3 Case (c)

Under the conditions (1.5c), we can find from expression (1.3) the following:

$$\begin{array}{r} a\,1\,7\,1 \\ \times\quad 1\,3\,1 \\ \hline a\,1\,7\,1 \\ +\quad i\,j\,k\,1\,3 \\ a\,1\,7\,1 \\ \hline 8\,q\,r\,4\,0\,1 \end{array} \qquad (1.6c)$$

Here the digital parameter k must be equal to 5 only, and the following two relations must be satisfied between parameters a, i, and j:

$$10 \times i + j = 3 \times a;\;\; a \geq 4;\;\; i > 0 \qquad (1.7c)$$

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$$7 \le a + i \le 8 \qquad (1.8c)$$

These two conditions (1.7c, 1.8c) can be satisfied only at a = 6; therefore, i = 1 and j = 8. Parameters r and q can be found by substituting all other known digits (a, i, j, and k) into expression (1.6c). We find r = 8 and q = 0. Finally, in this case, the expressions (1.1) and (1.6c) can be reduced to expression (1.9c) below:

$$\begin{array}{rr}
 & 6171 \\
\times & \underline{131} \\
 & 6171 \\
+ & 18513 \\
 & \underline{6171} \\
 & 808401
\end{array} \qquad (1.9c)$$

1.3 Answers:

a)

$$\begin{array}{rr}
 & 4191 \\
\times & \underline{191} \\
 & 4191 \\
+ & 37719 \\
 & \underline{4191} \\
 & 800481
\end{array}$$

b)

$$\begin{array}{rr}
 & 5131 \\
\times & \underline{171} \\
 & 5131 \\
+ & 35917 \\
 & \underline{5131} \\
 & 877401
\end{array}$$

b)

$$\begin{array}{rr}
 & 6171 \\
\times & \underline{131} \\
 & 6171 \\
+ & 18513 \\
 & \underline{6171} \\
 & 808401
\end{array}$$

NOTE: Only solution (a) was found in Ref. 12. ■

2. Study Case 2: Restoring the Digits in Arithmetic Calculations with Subtraction

2.1 Problem: Restore digits in the example below, where different letters indicate different digits and similar letters indicate similar digits [12]:

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   ANNA
–  KATE          (2.1)
   4 7 0 9
```

2.2 Solution: Following the condition that different letters indicate different digits, we find: N ≠ T; therefore, A ≠ 9, and N = T + 1 or **T = N – 1**. From this relationship between N and T, we can find that (10 + A) – E = 9 or **E = A + 1**. Further analysis shows that (10 + N) – A = 7 or **N = A – 3**; also, (A – 1) – K = 4 or **K = A – 5**, and A ≠ 5, otherwise, K = 0, which is impossible. Finally, the parameter **A** can only be **6**, **7**, and **8**. At these values for A, we find that only three cases are candidates for the solutions:

$$A = 6, K = 1, N = 3, T = 2, E = 7; \qquad (2.2a)$$

$$A = 7, K = 2, N = 4, T = 3, E = 8; \qquad (2.2b)$$

$$A = 8, K = 3, N = 5, T = 4, E = 9. \qquad (2.2c)$$

2.3 Answers:

```
a)  6336
–   1627
    4709

b)  7447
–   2738
    4709

c)  8558
–   3849
    4709
```

These answers (a, b, and c) were also found in Ref. 12. ■

3. Study Case 3: Evaluating Expressions with Infinite Square-Root Expressions

3.1 Problem: Find the value of the expression:

SQRT(a×SQRT(b×(SQRT(a×(SQRT(b×(SQRT(a×SQRT(b×(…)))))))))), (3.1)

where $a = 3^3$ and $b = 5^3$.

NOTE #1: There are an infinite number of terms in the above expression.
NOTE #2: SQRT stands for the *Square-Root* function, $\sqrt{\ }$.
NOTE #3: Several partial cases were considered in Ref. 12 at given parameters **a** and **b**. Here we offer a solution for the general case of arbitrary positive real numbers **a** and **b**.

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3.2 Solving Strategies

Two strategies will be discussed that can be explored by students.

3.2.1 The first strategy is based on the fact that there are an *infinite* number of terms in the given expression. Therefore, the equation (3.1) can be reformulated as the following:

$$X = SQRT(a\times SQRT(b\times(SQRT(a\times(SQRT(b\times(SQRT(a\times SQRT(b\times(\ldots)))))))))), \quad (3.2)$$

$$X = SQRT(a \times SQRT(b \times X)) \quad (3.3)$$

The last equation (3.3) can be easily solved:

$X^2 = a \times SQRT(b \times X)$; $X^4 = a^2\times b\times X$; $X^3 = a^2\times b$ and finally:

$$X = a^{2/3} \times b^{1/3} \quad (3.4)$$

Therefore, after substituting $a = 3^3 = 27$ and $b = 5^3 = 125$ into Eq. (3.4), we find that $X = 9 \times 5 = 45$, which is the answer to this problem.

3.2.2 A second strategy can be developed by using the concept of geometric progression. The expression (3.2) can be written as the following:

$$X = a^S \times b^T \quad (3.5)$$

where

$$S = ½ + ½ \times ¼ + ½ \times ¼ \times ¼ + \ldots + ½ \times (¼)^n + \ldots = ½ \div (1 - ¼) = ⅔ \quad (3.6)$$

$$T = ¼ + ¼ \times ¼ + ¼ \times ¼ \times ¼ + \ldots + ¼ \times (¼)^n + \ldots = ¼ \div (1 - ¼) = ⅓ \quad (3.7)$$

Therefore, $X = a^{2/3} \times b^{1/3}$, which is the equation (3.4).

3.3 Answer: $X = a^{2/3} \times b^{1/3}$. In the particular case, at $a = 3^3 = 27$ and $b = 5^3 = 125$, we find $X = 45$.■

4. Study Cases 4.1 – 4.3: Factor Analysis of Large Numbers

4.1 Problem: Prove that any six-digit number, which has the same three first and last digits (written in the same order), has factors (divisors) of 7, 11, and 13. The problem was offered in Ref. 12.

4.1.1 Solution to Problem 4.1. First of all, let's find the product of the factors, $7 \times 11 \times 13 = 1001$. Any six-digit number, which has the same three first and last digits (written in the same order), can be written in the following form:

$$abcabc = a \times 10^5 + b \times 10^4 + c \times 10^3 + a \times 10^2 + b \times 10^1 + c \quad (4.1a)$$

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$$abcabc = a \times 10^2 \times (10^3 + 1) + b \times 10^1 \times (10^3 + 1) + c \times (10^3 + 1) \quad (4.1b)$$

$$abcabc = 1001 \times (100 \times a + 10^1 \times b + c). \quad (4.1c)$$

Therefore, the six-digit number, **abcabc** can be divided by 1001 = 7 × 11 × 13, and numbers 7, 11, and 13 are the factors of the number, **abcabc**. ■

4.2 Problem: Prove that $(2^{1776} - 1)$ can be divided by 7.

4.2.1 Solution to Problem 4.2. It is easier to find that the power parameter 1776 can be divided by three, 1776 = 3 × 592. Therefore, the number 2^{1776} can be written as the following:

$$2^{1776} = 8^{592} \quad (4.2a)$$

Using the factoring rule, $A^2 - B^2 = (A - B) \times (A + B)$, we can find the following:

$$2^{1776}-1 = (8^{592}-1) = (8^{296}-1)\times(8^{296} + 1) = \ldots = (8^{37}-1)\times(8^{37}+1)\times(8^{74}+1)\times(8^{148}+1)\times(8^{296} + 1) \quad (4.2b)$$

At the next step, we can apply the Binomial Theorem to study the properties of the number 8^{37}:

$$(a + b)^n = a^n + n\times a^{n-1}\times b + n\times(n-1)/(2!)\times a^{n-2}\times b^2 + \ldots + n\times a\times b^{n-1} + b^n \quad (4.2c)$$

In our case, a = 7, b = 1, and n = 37, and the number 8^{37} can be written as the following:

$$8^{37} = (7 + 1)^{37} = 7^{37} + 37\times 7^{36}\times 1 + 37\times 36/(2)\times 7^{35}\times 1 + \ldots + 37\times 7\times 1 + 1 \quad (4.2d)$$

Therefore, the number $(8^{37}-1)$ can be divided by 7, and the original number $(2^{1776} - 1)$ can be divided by 7 as well. ■

4.3 Problem: Suppose **n** is an even natural number and **n** > 4. Prove that the number $(\mathbf{n}^5 - 20\mathbf{n}^3 + 64\mathbf{n})$ can be divided by 3840.

4.3.1 Solution to Problem 4.3. By factoring, it is easier to estimate that $3840 = 2^8 \times 3 \times 5$. Using the basic factoring rule, $A^2 - B^2 = (A - B) \times (A + B)$, twice, we can find the following:

$$n^5-20n^3+64n = n\times(n^4-20n^2+64) = n\times(n^2-16)\times(n^2-4) = (n-4)\times(n-2)\times n\times(n+2)\times(n+4) \quad (4.3a)$$

It is given that the parameter **n** is an even number and **n** > 4; therefore, we can introduce a new parameter k > 2, such that n = 2k. After substituting n = 2k to Eq. (4.3a), we find:

$$n^5-20n^3+64n = (2k-4)\times(2k-2)\times(2k)\times(2k+2)\times(2k+4) = 2^5\times(k-2)\times(k-1)\times k\times(k+1)\times(k+2) \quad (4.3b)$$

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The five consecutive numbers, (k–2), (k–1), k, (k+1), and (k+2) have numbers that can be divided by 2, 3, 4, and 5; therefore the product of these five numbers can be divided by $2^3 \times 3 \times 5$. Thus, the original number, $(\mathbf{n}^5 - 20\mathbf{n}^3 + 64\mathbf{n})$ can be divided by $2^8 \times 3 \times 5 = 3840$. ■

5. Study Case 5: Estimating the Last Digit of the Large Number

We start every class with a brief discussion of an unusual, non-trivial topic that is called a "warm-up" exercise [13]. After these "warm-up" exercises, the instructor offers a discussion about the main topic and asks students for feedback and for their suggestions for a competitive strategies with which to approach and analyze the problem. These discussions help students to focus on the main point of the class session and stay active in class. Here is an example of a "warm-up" exercise that opens an introductory discussion of the theory of large numbers, which leads to the applied theory of encryption algorithms, such as the RSA Public-Key encryption algorithm [14]. At the same time, it illustrates a strong bond between mathematics and computer science. A student (even if she/he is not familiar with the theory of numbers) can solve the problem by simple experimentation.

5.1 Problem: What is the Last Digit of the Number 2597^{5927} [mod(10)]?

5.2 Solution

We are interested in the last digit only of this number. Following Newton's Binomial Theorem (see section 4.2.1), it is enough to consider the last digit of a simpler number 7^{5927}. Let's do simple experiments with powers of number 7:

$$7^1 = \mathbf{7};\ 7^2 = 4\mathbf{9};\ 7^3 = 34\mathbf{3};\ 7^4 = 240\mathbf{1}; \tag{5.1}$$

$$7^5 = 16{,}80\mathbf{7};\ 7^6 = 117{,}64\mathbf{9};\ 7^7 = 823{,}54\mathbf{3};\ 7^4 = 5{,}764{,}80\mathbf{1};\ \textit{etc.} \tag{5.2}$$

We find that the last digit can only be 7, 9, 3, or 1, and therefore, there is a cycle of *four* cases. The power, 5927 can be represented as 5927 = 4×1481+3. Therefore, the last digit of 7^{5927} (and 2597^{5927}) is the same as the last digit of $7^3 = 343$, which is "**3**". Knowing two key parameters [e.g., the base (10) and power (5927)], we can now restore all digits of the huge number. ■

6. Study Case 6: Applying Simple Trigonometric Formulas

6.1 Problem: Estimate the value of the expression [12]:

$$\tan(1°) \times \tan(2°) \times \tan(3°) \times \ldots \times \tan(87°) \times \tan(88°) \times \tan(89°) \tag{6.1}$$

NOTE: There are 89 terms in this expression.

6.2 Solution

The following simple trigonometric identities can be used in the analysis:

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$$\tan(x) = \cot(90° - x), \text{ or } \tan(x) \times \tan(90° - x) = 1 \quad (6.2)$$

After regrouping all the terms in Eq. (6.1) in pairs and apply the Eq. (6.2) to each pair, we will find the following:

$$[\tan(1°) \times \tan(89°)] \times [\tan(2°) \times \tan(88°)] \times \ldots \times [\tan(44°) \times \tan(46°)] \times \tan(45°)$$
$$= 1 \times 1 \times \ldots \times 1 \times \tan(45°) = \tan(45°) = 1 \quad (6.3)$$

6.3 Answer:

$\tan(1°) \times \tan(2°) \times \tan(3°) \times \ldots \times \tan(87°) \times \tan(88°) \times \tan(89°) = \mathbf{1}$. ■

We invite readers to solve problems of study cases 7, 8, and 9, below, and share their solutions with others.

7. Study Case 7: "Minute" and "Hour" Clock-Arrows in Coupling Positions

7.1 Problem: How many times a day (24 hours) "minute" and "hour" clock-arrows take the same ("coupling") position? When does this coupling occur the first time? When does the coupling occur the seventh time?

8. Study Case 8: Logical Problem

8.1 Problem: In the following game, all students in the class were divided into two teams: "serious students", who were answering CORRECTLY to any question, and "jokers", who only were answering INCORRECTLY to any question. A teacher asked Smith is he a "serious" student or a "joker". The teacher did not hear Smith's answer well, and he asked both Parker and Dennis the question: "What did Smith answer?" Parker replied: "Smith's answer was that he is a 'serious student'." Dennis replied: "Smith's answer was that he is a 'joker'." For each of Parker and Dennis, determine if she/he is a "serious student" or a "joker".

9. Study Case 9: Combinatorics

9.1 Problem: The ice-hockey team includes three forwards, two defenders, and one goalkeeper. How many different teams the hockey-team trainer creates, if the Rivier College team has 7 forwards, 5 defenders, and 2 goalkeepers?

10. Conclusion

Nine specially-designed study cases have been used for motivating students (both in high schools and colleges) to explore the advanced topics of Elementary Mathematics and Algebra. This approach allows the students to build strong analytical skills and search for modern applications of these topics in various areas of mathematics and computer science.

ISSN 1559-9388 (online version), ISSN 1559-9396 (CD-ROM version).

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ISSN 1559-9388 (online version), ISSN 1559-9396 (CD-ROM version).